

出版前言

2011年12月11日是西安交通大学杰出校友钱学森先生的百年诞辰。为缅怀钱学森学长,学习他的科学思想和卓越风范,展示其丰功伟绩和人格魅力,西安交通大学举办了“纪念钱学森诞辰100周年”系列活动:作为制片方之一,参与西部电影集团摄制传记故事片《钱学森》;与中央电视台合作,出品纪录片《实验班的故事——沿着钱学森走过的路》;扩建钱学森生平业绩展馆,向校内外开放;举办钱学森科学与教育思想研讨会;出版发行《钱学森力学手稿》、《钱学森年谱(初编)》、《钱学森第六次产业革命思想探微丛书》等。

钱学森先生在美国深造和工作期间留下大量珍贵手稿,这些手稿真实展示了钱学森先生博大精深的学识、开拓求实的精神和严谨奋进的作风,是钱老勇攀科学高峰和严谨治学的集中体现。这里,我们将部分原稿整理汇集成册,出版《钱学森力学手稿》,作为钱老百年诞辰的献礼。

《钱学森力学手稿》共10卷,包含两部分内容。第一部分是草稿,包括扁壳、球壳和圆柱壳屈曲分析的公式推导和数值演算。在研究圆柱壳轴压屈曲问题时,为了求得圆柱壳体的临界压力,在有关的五百多页草稿中,对多达二十多种可能的屈曲模

态逐一进行公式推演和数值计算,最终才找到满意的并在论文中采用的屈曲模态。仔细观察草稿中的数据列表,每个数字有效位数都长达八位,在手摇机械式计算机作为主要计算工具的年代,这串串数字凝聚着多少现今难以想象的艰辛劳动。

第二部分是手稿,以航空航天工程为核心,涵盖空气动力学、固体力学、火箭技术、工程控制论和物理力学等领域的部分学术论文手稿、打印稿和讲义。

《钱学森力学手稿》是在西安交通大学校领导的大力支持下,由西安交通大学航天航空学院沈亚鹏教授整理完成。图书出版过程中得到了西安交通大学党委宣传部、校友关系发展部、图书馆、航天航空学院等的积极协助,在此深表感谢。

Preliminary Calculation of
Circular Cylinder (IV)

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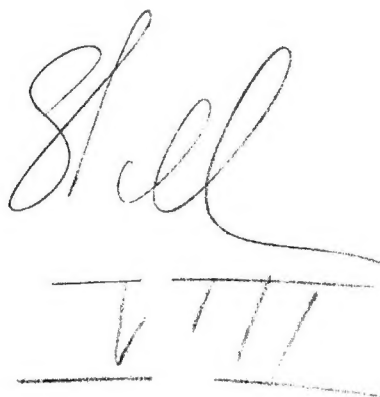
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Asst.

We have the equation

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$$\frac{\partial^2 \psi}{\partial x^2} + 2 \frac{\partial^2 \psi}{\partial y^2} = - \left\{ \frac{\partial \psi}{\partial y} \left(\frac{\partial^2 \psi}{\partial x^2} + 2 \frac{\partial^2 \psi}{\partial y^2} \right) + \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial x \partial y} \right\}$$

$$\frac{\psi}{R} = \frac{1}{2} \left(\frac{a}{R} \right)^2 \left[1 - \left(\frac{x}{a} \right)^2 - \frac{f}{2} \cos \frac{\pi x}{2a} (1 + \cos \frac{\pi y}{b}) \right]$$

$$\frac{\psi_0}{R} = \frac{1}{2} \left(\frac{a}{R} \right)^2 \left[1 - \left(\frac{x}{a} \right)^2 \right]$$

$$\frac{\partial \psi}{\partial x} = \left(\frac{a}{R} \right) \left[- \left(\frac{x}{a} \right) + \frac{f}{8} \pi \sin \frac{\pi x}{2a} (1 + \cos \frac{\pi y}{b}) \right]$$

$$\frac{\partial \psi}{\partial y} = \left(\frac{a}{R} \right) \left[+ \frac{f}{4} \pi \lambda \cos \frac{\pi x}{2a} \sin \frac{\pi y}{b} \right]$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{R} \left[-1 + \frac{f}{16} \pi^2 \cos \frac{\pi x}{2a} (1 + \cos \frac{\pi y}{b}) \right]$$

$$\frac{\partial^2 \psi}{\partial x \partial y} = \frac{1}{R} \left[- \frac{f}{8} \pi^2 \lambda \sin \frac{\pi x}{2a} \sin \frac{\pi y}{b} \right]$$

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{1}{R} \left[+ \frac{f}{4} \pi^2 \lambda^2 \cos \frac{\pi x}{2a} \cos \frac{\pi y}{b} \right]$$

$$R \left[\frac{\partial^2 \psi}{\partial x^2} + 2 \frac{\partial^2 \psi}{\partial y^2} \right] = - \left(\frac{a}{R} \right) \left(\frac{f}{4} \pi \lambda \right) \left[\cos \frac{\pi x}{2a} \sin \frac{\pi y}{b} \left\{ -1 + \frac{f \pi^2}{16} \cos \frac{\pi x}{2a} (1 + \cos \frac{\pi y}{b}) \right\} \right. \\ \left. + \frac{f \pi^2}{2} \lambda^2 \cos \frac{\pi x}{2a} \cos \frac{\pi y}{b} \right]$$

$$- \frac{\pi}{2} \sin \frac{\pi x}{2a} \sin \frac{\pi y}{b} \left\{ - \left(\frac{x}{a} \right) + \frac{f \pi}{8} \sin \frac{\pi x}{2a} (1 + \cos \frac{\pi y}{b}) \right\} \Big]$$

$$= - \left(\frac{a}{R} \right) \frac{f}{4} \pi \lambda \left[- \cos \frac{\pi x}{2a} \sin \frac{\pi y}{b} + \frac{f \pi^2}{64} (1 + \cos \frac{\pi x}{a}) (2 \sin \frac{\pi y}{b} + \sin \frac{2 \pi y}{b}) \right. \\ \left. + \left(\frac{\pi x}{2a} \right) \sin \frac{\pi x}{2a} \sin \frac{\pi y}{b} + \frac{f \pi^2}{64} (-1 + \cos \frac{\pi x}{a}) (2 \sin \frac{\pi y}{b} + \sin \frac{2 \pi y}{b}) \right. \\ \left. + \frac{f \pi^2}{8} \lambda^2 (1 + \cos \frac{\pi x}{a}) \sin \frac{2 \pi y}{b} \right]$$

$$\begin{aligned}
 \nabla^2 \left[\frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 u}{\partial y^2} \right] &= - \left(\frac{g}{b} \right) \frac{\pi \lambda^2}{4} \left[\left(\frac{\pi \lambda}{ga} \right) \sin \frac{\pi x}{2a} \sin \frac{\pi y}{b} - \cos \frac{\pi x}{2a} \sin \frac{\pi y}{b} + \frac{\pi^2}{16} \cos \frac{\pi x}{a} \sin \frac{\pi y}{b} \right. \\
 &\quad \left. + \frac{\pi^2}{32} \cos \frac{\pi x}{a} \sin \frac{2\pi y}{b} + \frac{\pi^2}{8} \lambda^2 \sin \frac{2\pi y}{b} + \frac{\pi^2}{8} \lambda^2 \cos \frac{\pi x}{a} \sin \frac{2\pi y}{b} \right] \\
 &= - \left(\frac{g}{b} \right) \frac{\pi \lambda^2}{4} \left[\left(\frac{\pi \lambda}{ga} \right) \sin \frac{\pi x}{2a} \sin \frac{\pi y}{b} - \cos \frac{\pi x}{2a} \sin \frac{\pi y}{b} + \frac{\pi^2}{8} \lambda^2 \sin \frac{2\pi y}{b} + \frac{\pi^2}{16} (1 + 2\lambda^2) \cos \frac{\pi x}{a} \sin \frac{\pi y}{b} \right. \\
 &\quad \left. + \frac{\pi^2}{32} \cos \frac{\pi x}{a} \sin \frac{2\pi y}{b} \right]
 \end{aligned}$$

Investigate the particular solution of above following eqn.

$$\frac{\partial^2 v}{\partial x^2} + 2 \frac{\partial^2 v}{\partial y^2} = \left(\frac{\pi \lambda}{ga} \right) \sin \frac{\pi x}{2a} \sin \frac{\pi y}{b}$$

Put

$$v = X \sin \frac{\pi y}{b}$$

$$\frac{d^2 X}{dx^2} - 2 \left(\frac{\pi}{b} \right)^2 X = \left(\frac{\pi}{ga} \right) \sin \frac{\pi x}{2a}$$

Let

$$X = A \cos \frac{\pi x}{2a} + B \left(\frac{\pi x}{2a} \right) \sin \frac{\pi x}{2a}$$

$$\frac{d^2 X}{dx^2} = \left(\frac{\pi}{2a} \right)^2 \left(2B - A \right) \cos \frac{\pi x}{2a} - \frac{B}{4} \left(\frac{\pi}{2a} \right) \sin \frac{\pi x}{2a}$$

$$- 2 \left(\frac{\pi}{b} \right)^2 X = \left(\frac{\pi}{a} \right)^2 \left\{ - 2\lambda^2 A \cos \frac{\pi x}{2a} - 2\lambda^2 B \left(\frac{\pi x}{2a} \right) \sin \frac{\pi x}{2a} \right\}$$

$$\therefore \left(\frac{\pi}{a} \right)^2 \left\{ \frac{2B - A}{4} - 2\lambda^2 A \right\} = 0, \quad \left(\frac{\pi}{a} \right)^2 \left\{ - \frac{B}{4} - 2\lambda^2 B \right\} = 1$$

$$\begin{aligned} \frac{v}{R} = & \left(\frac{a}{R} \right)^3 \frac{1}{4} \pi \lambda \left[\frac{2}{(1+h^2)^2} \cos \frac{\pi x}{2a} \sin \frac{\pi y}{b} + \frac{4}{1+h^2} \left(\frac{\pi x}{2a} \right) \sin \frac{\pi x}{2a} \sin \frac{\pi y}{b} - \frac{4}{(1+h^2)} \cos \frac{\pi x}{2a} \sin \frac{\pi y}{b} \right. \\ & + \frac{h^2}{8} \lambda^2 \frac{1}{b^2} \sin^2 \frac{\pi y}{b} + \frac{h^2}{16} \frac{1+h^2}{1+h^2} \cos \frac{\pi x}{a} \sin \frac{\pi y}{b} + \frac{h^2}{32} \frac{1}{1+h^2} \cos \frac{\pi x}{a} \sin \frac{2\pi y}{b} \\ & \left. + a_0 \left(\frac{\pi y}{b} \right) + a_2 \cos \sqrt{2} \frac{\pi \lambda x}{a} \sin \frac{\pi y}{b} \right] \end{aligned}$$

$$\begin{aligned} = & \left(\frac{a}{R} \right)^3 \frac{1}{4} \pi \lambda \left[\frac{4(1-h^2)}{(1+h^2)^2} \cos \frac{\pi x}{2a} \sin \frac{\pi y}{b} + \frac{4}{1+h^2} \left(\frac{\pi x}{2a} \right) \sin \frac{\pi x}{2a} \sin \frac{\pi y}{b} + \frac{h^2}{64} \sin \frac{2\pi y}{b} \right. \\ & \left. + \frac{h^2}{16} \cos \frac{\pi x}{a} \sin \frac{\pi y}{b} + \frac{h^2}{32} \frac{1}{(1+h^2)} \cos \frac{\pi x}{a} \sin \frac{2\pi y}{b} + a_0 \left(\frac{\pi y}{b} \right) + a_2 \cos \sqrt{2} \frac{\pi \lambda x}{a} \sin \frac{\pi y}{b} \right] \end{aligned}$$

$$\begin{aligned} \frac{\partial v}{\partial x} = & \left(\frac{a}{R} \right)^3 \frac{1}{4} \pi \lambda \left[- \frac{2(1-h^2)}{(1+h^2)^2} \sin \frac{\pi x}{2a} \sin \frac{\pi y}{b} + \frac{2}{1+h^2} \sin \frac{\pi x}{2a} \sin \frac{\pi y}{b} + \frac{2}{(1+h^2)} \left(\frac{\pi x}{2a} \right) \cos \frac{\pi x}{2a} \sin \frac{\pi y}{b} \right. \\ & \left. - \frac{h^2}{16} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} - \frac{h^2}{32} \frac{1}{(1+h^2)} \sin \frac{\pi x}{a} \sin \frac{2\pi y}{b} + a_2 \sqrt{2} \lambda \sin \sqrt{2} \frac{\pi \lambda x}{a} \sin \frac{\pi y}{b} \right] \end{aligned}$$

$$\frac{\partial v}{\partial x} \frac{\partial w}{\partial y} = \left(\frac{a}{R} \right)^2 \frac{1}{4} \pi \lambda \left[- 2 \left(\frac{\pi x}{2a} \right) \cos \frac{\pi x}{2a} \sin \frac{\pi y}{b} + \frac{h^2}{32} \sin \frac{\pi x}{a} \left(2 \sin \frac{\pi y}{b} + \sin \frac{2\pi y}{b} \right) \right]$$

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$$\begin{aligned} \left(\frac{a}{R}\right)^2 \frac{1}{4} \lambda^2 a_0 &= -\frac{\sigma}{E} - \left(\frac{a}{R}\right)^2 \frac{1}{4} \lambda^2 \left[-\frac{16}{\pi} \frac{1}{(1+\lambda)^2} + \frac{16}{32} + \frac{16}{\pi} \frac{1}{(1+\lambda)^2} \right] \\ &= -\frac{\sigma}{E} - \left(\frac{a}{R}\right)^2 \frac{1}{4} \lambda^2 \left(\frac{16}{32} \right) \end{aligned}$$

$$\boxed{\frac{\Delta \rho}{a b t E} = -\left(\frac{\sigma}{E}\right)^2 - \left(\frac{a}{R}\right)^2 \left(\frac{1}{4}\right)^2 \left(\frac{\pi^2 \lambda^2}{8}\right)}$$

$$\varepsilon_1 = \frac{1}{2} \left\{ \left(\frac{\partial u}{\partial x} \right)^2 - \left(\frac{\partial u}{\partial y} \right)^2 \right\} = \left(\frac{a}{R} \right)^2 \frac{1}{4} \left\{ -\left(\frac{\pi}{2a} \right) \sin \frac{\pi x}{2a} \left(1 + \cos \frac{\pi x}{a} \right) + \frac{16}{128} \left(1 - \cos \frac{\pi x}{a} \right) \left(3 + 4 \cos \frac{\pi x}{a} + \cos \frac{2\pi x}{a} \right) \right\}$$

$$= \left(\frac{a}{R} \right)^2 \frac{1}{4} \left\{ -\left(\frac{\pi}{2a} \right) \sin \frac{\pi x}{2a} + \frac{3 \pi^2}{128} \left(1 - \cos \frac{\pi x}{a} \right) \right\}$$

$$+ \left\{ -\left(\frac{\pi}{2a} \right) \sin \frac{\pi x}{2a} + \frac{16}{32} \left(1 - \cos \frac{\pi x}{a} \right) \cos \frac{\pi x}{a} \right\}$$

$$+ \frac{16}{128} \left(1 - \cos \frac{\pi x}{a} \right) \cos \frac{2\pi x}{a} \left. \right\}$$

$$\frac{1}{2ab} \int_0^a \int_0^b \varepsilon_1^2 dx dy = \left(\frac{a}{R} \right)^4 \left(\frac{1}{4} \right)^2 \left\{ \frac{3}{4} \left[\left(\frac{3}{128} \right)^2 + \frac{1}{2} \left(\frac{1}{32} \right)^2 + \frac{1}{2} \left(\frac{1}{128} \right)^2 \right] \left(\frac{\pi^2}{4} \right)^2 + \frac{5}{4} \int_0^1 \left(\frac{\pi x}{2a} \right)^2 \sin^2 \frac{\pi x}{2a} dx \right\}$$

$$- \frac{5}{128} \left(\frac{\pi^2}{4} \right) \int_0^1 \left(\frac{\pi x}{2a} \right) \sin \frac{\pi x}{2a} \left(1 - \cos \frac{\pi x}{a} \right) dx \left. \right\}$$

$$= \left(\frac{a}{R} \right)^4 \left(\frac{1}{4} \right)^2 \left[\frac{105}{8(128)} \left(\frac{\pi^2}{4} \right)^2 - \frac{5}{128} \left(\frac{3}{4} + \frac{10}{9\pi} \right) \left(\frac{\pi^2}{4} \right) + \left(\frac{\pi^2}{32} + \frac{3}{16} \right) \right]$$

$$\frac{\lambda^2}{\epsilon_0} = \left(\frac{q}{R}\right)^2 \frac{1}{4} \lambda^2 \left[\frac{4(1-\delta\lambda^2)}{(1+\delta\lambda^2)^2} \cos \frac{\pi x}{2a} \cos \frac{\pi y}{b} + \frac{4}{(1+\delta\lambda^2)^2} \left(\frac{\pi x}{2a}\right) \sin \frac{\pi x}{2a} \cos \frac{\pi y}{b} + \frac{4\pi^2}{32} \cos \frac{2\pi y}{b} \right. \\ \left. + \frac{4\pi^2}{16} \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} + \frac{4\pi^2}{16} \frac{1}{(1+\delta\lambda^2)} \cos \frac{\pi x}{a} \cos \frac{2\pi y}{b} - \frac{4\pi^2}{32} + a_2 \cos \left(\sqrt{2} \frac{\pi \lambda x}{a} \right) \cos \frac{\pi y}{b} \right] - \frac{\sigma}{\epsilon}$$

$$\frac{1}{2} \left(\frac{\lambda \omega}{2\pi} \right)^2 = \left(\frac{q}{R} \right)^2 \frac{1}{4} \lambda^2 \left[\frac{4\pi^2}{32} (1 + \cos \frac{\pi x}{a}) (1 - \cos \frac{2\pi y}{b}) \right] = \left(\frac{q}{R} \right)^2 \frac{1}{4} \lambda^2 \left[\frac{4\pi^2}{32} + \frac{4\pi^2}{32} \cos \frac{\pi x}{a} - \frac{4\pi^2}{32} \cos \frac{2\pi y}{b} \right. \\ \left. - \frac{4\pi^2}{32} \cos \frac{\pi x}{a} \cos \frac{2\pi y}{b} \right]$$

$$\epsilon_2 = \left(\frac{q}{R} \right)^2 \frac{1}{4} \lambda^2 \left[\frac{4(1-\delta\lambda^2)}{(1+\delta\lambda^2)^2} \cos \frac{\pi x}{2a} \cos \frac{\pi y}{b} + \frac{4}{(1+\delta\lambda^2)^2} \left(\frac{\pi x}{2a} \right) \sin \frac{\pi x}{2a} \cos \frac{\pi y}{b} + \frac{4\pi^2}{16} \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} + \frac{4\pi^2}{32} \cos \frac{2\pi y}{b} \right. \\ \left. + \frac{4\pi^2}{32} \left(\frac{1-\delta\lambda^2}{(1+\delta\lambda^2)} \right) \cos \frac{\pi x}{a} \cos \frac{2\pi y}{b} + a_2 \cos \left(\sqrt{2} \frac{\pi \lambda x}{a} \right) \cos \frac{\pi y}{b} \right] - \frac{\sigma}{\epsilon}$$

$$= \left(\frac{q}{R} \right)^2 \frac{1}{4} \lambda^2 \left[\frac{4\pi^2}{32} \cos \frac{\pi x}{a} \right. \\ \left. + \left\{ \frac{4(1-\delta\lambda^2)}{(1+\delta\lambda^2)^2} \cos \frac{\pi x}{2a} + \frac{4\pi^2}{16} \cos \frac{\pi x}{a} + \frac{4}{(1+\delta\lambda^2)^2} \left(\frac{\pi x}{2a} \right) \sin \frac{\pi x}{2a} + a_2 \cos \left(\sqrt{2} \frac{\pi \lambda x}{a} \right) \right\} \cos \frac{\pi y}{b} \right. \\ \left. + \left\{ \frac{4\pi^2}{32} \left(\frac{1-\delta\lambda^2}{(1+\delta\lambda^2)} \right) \cos \frac{\pi x}{a} + \frac{4\pi^2}{32} \cos \frac{2\pi y}{b} \right\} \right] - \frac{\sigma}{\epsilon}$$

$$\begin{aligned}
\frac{1}{g_{ab}} \int_0^a \int_0^b \varepsilon^2 dx dy &= \left(\frac{a}{R}\right)^4 \left(\frac{b}{4}\right)^2 \lambda^4 \left[\frac{1}{4} \left(\frac{b\lambda^2}{32}\right)^2 + \frac{1}{8} \left\{ \frac{4(1-b\lambda^2)}{(1+b\lambda^2)^2} \right\}^2 + \frac{1}{8} \left(\frac{b\lambda^2}{16}\right)^2 \right] \\
&+ \frac{2(1+b\lambda^2)}{(1+b\lambda^2)^2} \int_0^1 \cos \frac{\pi x}{2a} \left\{ -\frac{4}{1+b\lambda^2} \left(\frac{\pi x}{2a}\right) \sin \frac{\pi x}{2a} + g_2 \cosh \frac{\sqrt{2}\pi \lambda x}{a} \right\} d\left(\frac{x}{a}\right) \\
&+ \frac{b\lambda^2}{32} \int_0^1 \cos \frac{\pi x}{a} \left\{ \frac{4}{1+b\lambda^2} \left(\frac{\pi x}{2a}\right) \sin \frac{\pi x}{2a} + g_2 \cosh \frac{\sqrt{2}\pi \lambda x}{a} \right\} d\left(\frac{x}{a}\right) \\
&+ \frac{1}{4} \int_0^1 \left\{ \frac{4}{1+b\lambda^2} \left(\frac{\pi x}{2a}\right) \sin \frac{\pi x}{2a} + g_2 \cosh \frac{\sqrt{2}\pi \lambda x}{a} \right\}^2 d\left(\frac{x}{a}\right) \\
&+ \frac{1}{8} \left\{ \frac{b\lambda^2}{32} \frac{1-b\lambda^2}{1+b\lambda^2} \right\}^2 + \frac{1}{2} \left(\frac{\sigma}{E}\right)^2
\end{aligned}$$

$$\begin{aligned}
\frac{1}{4ab} \int_0^a \int_0^b dx dy &= \left(\frac{a}{R}\right)^2 \left(\frac{b}{4}\right)^2 \lambda^2 \left[\frac{1}{4} \left\{ \frac{16b^2}{(1+b\lambda^2)^2} \right\}^2 + \frac{16b^2}{(1+b\lambda^2)^2} \int_0^1 \sin \frac{\pi x}{2a} \left\{ -\frac{b\lambda^2}{1+b\lambda^2} \left(\frac{\pi x}{2a}\right) \cos \frac{\pi x}{2a} + \frac{g_2 \lambda}{\sqrt{2}} \sinh \frac{\sqrt{2}\pi \lambda x}{a} \right\} d\left(\frac{x}{a}\right) \right. \\
&+ \frac{1}{2} \int_0^1 \left\{ -\frac{b\lambda^2}{(1+b\lambda^2)} \left(\frac{\pi x}{2a}\right) \cos \frac{\pi x}{2a} + \frac{g_2 \lambda}{\sqrt{2}} \sinh \frac{\sqrt{2}\pi \lambda x}{a} \right\}^2 d\left(\frac{x}{a}\right) \\
&\left. + \frac{1}{4} \left(\frac{b\lambda^2}{8} \frac{b^2}{1+b\lambda^2} \right)^2 \right]
\end{aligned}$$

$$\int_0^1 \left(\frac{\pi x}{2a} \right) \sin \frac{\pi x}{2a} \cos \frac{\pi x}{2a} d\left(\frac{x}{a}\right) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \theta \sin \theta d\theta = \frac{1}{4\pi} \int_0^{\pi} x \sin x dx = \frac{1}{4\pi} \left[\sin x - x \cos x \right]_0^{\pi} = \underline{\underline{\frac{1}{4}}}$$

$$\begin{aligned} \int_0^1 \frac{\cos \frac{\pi x}{2a} \cosh \frac{\sqrt{2}\pi x}{a} d\left(\frac{x}{a}\right)}{2\sqrt{2}\lambda + i} &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left[\cosh(\sqrt{2}\lambda + i)\theta + \cosh(2\sqrt{2}\lambda - i)\theta \right] d\theta \\ &= \frac{1}{\pi} \left[\frac{\sinh(2\sqrt{2}\lambda + i)\frac{\pi}{2}}{2\sqrt{2}\lambda + i} + \frac{\sinh(2\sqrt{2}\lambda - i)\frac{\pi}{2}}{2\sqrt{2}\lambda - i} \right] = \frac{2}{\pi} \frac{\cosh \sqrt{2}\lambda \pi}{(1 + \theta^2)} \end{aligned}$$

$$\begin{aligned} \int_0^1 \left(\frac{\pi x}{2a} \right) \sin \frac{\pi x}{2a} \cos \frac{\pi x}{a} d\left(\frac{x}{a}\right) &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \theta [\sin 3\theta - \sin \theta] d\theta = \frac{1}{\pi} \left[\frac{1}{9} \{ \sin 3\theta - 3\theta \cos \theta \} - \{ \sin \theta - \theta \cos \theta \} \right]_0^{\frac{\pi}{2}} \\ &= -\underline{\underline{\frac{10}{9\pi}}} \end{aligned}$$

$$\begin{aligned} \int_0^1 \frac{\cosh \frac{\sqrt{2}\pi \lambda x}{a} \cos \frac{\pi x}{a} d\left(\frac{x}{a}\right)}{\sqrt{2}\lambda + i} &= \frac{1}{2\pi} \int_0^{\frac{\pi}{2}} \left[\cosh(\sqrt{2}\lambda + i)\theta + \cosh(\sqrt{2}\lambda - i)\theta \right] d\theta \\ &= \frac{1}{2\pi} \left[\frac{\sinh(\sqrt{2}\lambda + i)\pi}{\sqrt{2}\lambda + i} + \frac{\sinh(\sqrt{2}\lambda - i)\pi}{\sqrt{2}\lambda - i} \right] = -\frac{1}{\pi} \frac{\sqrt{2}\lambda \sinh \sqrt{2}\lambda \pi}{1 + \lambda^2} \end{aligned}$$

$$\begin{aligned} \int_0^1 \left(\frac{\pi x}{2a} \right)^2 \sin \frac{\pi x}{2a} \cos \frac{\pi x}{a} d\left(\frac{x}{a}\right) &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \theta^2 [1 - \cos 2\theta] d\theta = \frac{1}{\pi} \left[\frac{\theta^3}{3} (2) - \frac{1}{8} \{ 2\lambda \cos \lambda + (\lambda^2 - 2) \sin \lambda \} \right]_0^{\frac{\pi}{2}} \\ &= \frac{1}{\pi} \left[\frac{1}{24} \pi^3 + \frac{1}{4} \pi \right] = \underline{\underline{\left[\frac{\pi^2}{24} + \frac{1}{4} \right]}} \end{aligned}$$

$$\begin{aligned}
 \int_0^1 \frac{\left(\frac{\pi x}{2a}\right) \sin \frac{\pi x}{2a} \cosh \frac{\sqrt{2} \pi \lambda x}{a} d\left(\frac{x}{a}\right)}{\pi i} &= \frac{1}{\pi i} \int_0^{\frac{\pi}{2}} \theta \left[\sinh(2\sqrt{2}\lambda + i)\theta - \sinh(2\sqrt{2}\lambda - i)\theta \right] d\theta \\
 &= \frac{1}{\pi i} \left[\frac{\frac{\pi}{2}}{2\sqrt{2}\lambda + i} \cosh(2\sqrt{2}\lambda + i)\frac{\pi}{2} - \frac{\frac{\pi}{2}}{2\sqrt{2}\lambda - i} \cosh(2\sqrt{2}\lambda - i)\frac{\pi}{2} \right. \\
 &\quad \left. - \left\{ \frac{1}{(2\sqrt{2}\lambda + i)^2} \sinh(2\sqrt{2}\lambda + i)\frac{\pi}{2} - \frac{1}{(2\sqrt{2}\lambda - i)^2} \sinh(2\sqrt{2}\lambda - i)\frac{\pi}{2} \right\} \right] \\
 &= \frac{1}{\pi} \left[\frac{2\sqrt{2}\pi\lambda \sinh \sqrt{2}\lambda \pi}{1 + \lambda^2} + \frac{2(1 - \lambda^2) \cosh \sqrt{2}\lambda \pi}{(1 + \lambda^2)^2} \right]
 \end{aligned}$$

$$\int_0^1 \cosh^2 \frac{\sqrt{2} \pi \lambda x}{a} d\left(\frac{x}{a}\right) = \frac{1}{2} \int_0^1 \left[1 + \cosh \frac{2\sqrt{2} \pi \lambda x}{a} \right] d\left(\frac{x}{a}\right) = \frac{1}{2} + \frac{\sinh 2\sqrt{2}\lambda \pi}{4\sqrt{2}\pi\lambda}$$

$$\int_0^1 \frac{\left(\frac{\pi x}{2a}\right) \sin \frac{\pi x}{2a} \cosh \frac{\sqrt{2} \pi \lambda x}{2a} d\left(\frac{x}{a}\right)}{\pi i} = \frac{1}{4}$$

$$\begin{aligned}
 \int_0^1 \frac{\left(\frac{\pi x}{2a}\right) \sin \frac{\pi x}{2a} \cosh \frac{\sqrt{2} \pi \lambda x}{a} d\left(\frac{x}{a}\right)}{\pi i} &= \frac{1}{\pi i} \int_0^{\frac{\pi}{2}} \left[\cosh(2\sqrt{2}\lambda + i)\theta - \cosh(2\sqrt{2}\lambda - i)\theta \right] d\theta \\
 &= \frac{1}{\pi i} \left[\frac{\sinh(2\sqrt{2}\lambda + i)\frac{\pi}{2}}{2\sqrt{2}\lambda + i} - \frac{\sinh(2\sqrt{2}\lambda - i)\frac{\pi}{2}}{2\sqrt{2}\lambda - i} \right] = \frac{1}{\pi} \frac{4\sqrt{2}\lambda \cosh \sqrt{2}\lambda \pi}{(1 + \lambda^2)^2}
 \end{aligned}$$

$$\int_0^1 \frac{\left(\frac{\pi x}{2a}\right)^2 \cos \frac{\pi x}{2a} d\left(\frac{x}{a}\right)}{\pi} = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \theta^2 [1 + \cos 2\theta] d\theta = \left[\frac{\pi^2}{24} - \frac{1}{4} \right]$$

$$\begin{aligned}
\int_0^1 \frac{\pi}{2a} \coth \frac{\pi}{2a} \sinh \frac{\sqrt{2}\pi\lambda x}{a} d\left(\frac{x}{a}\right) &= \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \sinh(2\sqrt{2}\lambda + i)\theta + \sinh(2\sqrt{2}\lambda - i)\theta \, d\theta \\
&= \frac{1}{\pi} \left[\frac{\frac{\pi}{2}}{2\sqrt{2}\lambda + i} \coth(2\sqrt{2}\lambda + i)\frac{\pi}{2} + \frac{\frac{\pi}{2}}{2\sqrt{2}\lambda - i} \coth(2\sqrt{2}\lambda - i)\frac{\pi}{2} \right] \\
&\quad - \frac{1}{\pi} \left[\frac{1}{(2\sqrt{2}\lambda + i)^2} \sinh(2\sqrt{2}\lambda + i)\frac{\pi}{2} + \frac{1}{(2\sqrt{2}\lambda - i)^2} \sinh(2\sqrt{2}\lambda - i)\frac{\pi}{2} \right] \\
&= \frac{1}{\pi} \left[\frac{\pi \sinh \sqrt{2}\pi\lambda}{1 + \lambda^2} - \frac{\sinh \sqrt{2}\pi\lambda \coth \sqrt{2}\pi\lambda}{(1 + \lambda^2)^2} \right]
\end{aligned}$$

$$\int_0^1 \sinh \frac{\sqrt{2}\pi\lambda x}{a} d\left(\frac{x}{a}\right) = \frac{\sinh 2\sqrt{2}\pi\lambda}{4\sqrt{2}\pi\lambda} - \frac{1}{2}$$

$$\begin{aligned}
\frac{1}{246} \int_0^{\pi/2} \varepsilon^2 dx dy &= \left(\frac{2}{15} \right) \left(\frac{1}{4} \right)^2 \lambda^4 \left[\frac{1}{4} \left(\frac{\lambda^2}{32} \right)^2 + 2 \frac{(1-\delta\lambda^2)^2}{(1+\delta\lambda^2)^4} + \frac{1}{8} \left(\frac{\lambda^2}{16} \right)^2 \right] \\
&+ \frac{2(1-\delta\lambda^2)}{(1+\delta\lambda^2)^2} \left\{ \frac{1}{16\lambda^2} - \frac{64\lambda^2 \cos \sqrt{2}\lambda\pi}{(1+\delta\lambda^2)^3 (\sqrt{2}\lambda\pi \sin \sqrt{2}\lambda\pi)} \right\} + \frac{\lambda^2}{32} \left\{ -\frac{42}{9\pi} \frac{1}{(1+\delta\lambda^2)} + \frac{32\lambda^2}{(1+\delta\lambda^2)^3 (1+\delta\lambda^2)^2 \pi} \right\} \\
&+ \frac{1}{4} \left\{ \left(\frac{\pi^2}{24} + \frac{1}{4} \right) \frac{16}{(1+\delta\lambda^2)^2} - \frac{2}{1+\delta\lambda^2} \frac{32\lambda^2}{(1+\delta\lambda^2)^2} \left\{ \frac{2}{1+\delta\lambda^2} + \frac{2(1-\delta\lambda^2)}{(1+\delta\lambda^2)^2} \frac{\cos \sqrt{2}\lambda\pi}{(\sqrt{2}\lambda\pi \sin \sqrt{2}\lambda\pi)} \right\} \right\} \\
&+ \frac{1}{4} \frac{(32\lambda^2)^2}{(1+\delta\lambda^2)^4 (\sqrt{2}\lambda \sin \sqrt{2}\lambda\pi)^2} \left\{ \frac{1}{2} + \frac{\sin \sqrt{2}\lambda\pi}{4\sqrt{2}\lambda\pi} \right\} + \frac{1}{8} \left\{ \frac{-\lambda^2}{32} \frac{1-\delta\lambda^2}{1+\delta\lambda^2} \right\}^2 + \frac{1}{2} \left(\frac{\sigma}{E} \right)^2
\end{aligned}$$

$$\begin{aligned}
\frac{1}{496} \int_0^{\pi/2} \delta^2 dx dy &= \left(\frac{2}{15} \right) \left(\frac{1}{4} \right)^2 \lambda^2 \left[-\frac{64\lambda^4}{(1+\delta\lambda^2)^4} + \frac{16\lambda^2}{(1+\delta\lambda^2)^2} \right] - \frac{2\lambda^2}{1+\delta\lambda^2} - \frac{128\lambda^4 \cos \sqrt{2}\lambda\pi}{(1+\delta\lambda^2)^3 (\sqrt{2}\lambda\pi \sin \sqrt{2}\lambda\pi)} \left\{ \right. \\
&+ \frac{1}{2} \frac{64\lambda^4}{(1+\delta\lambda^2)^2} \left(\frac{\pi^2}{24} - \frac{1}{4} \right) + \frac{128\lambda^4}{(1+\delta\lambda^2)^3} \left\{ \frac{1}{(1+\delta\lambda^2)} - \frac{16\lambda^2 \cos \sqrt{2}\lambda\pi}{(1+\delta\lambda^2)^3 (\sqrt{2}\lambda\pi \sin \sqrt{2}\lambda\pi)} \right\} \\
&\left. + \frac{\lambda^2}{4} \frac{(32\lambda^2)^2}{(1+\delta\lambda^2)^4 (\sqrt{2}\lambda \sin \sqrt{2}\lambda\pi)^2} \left\{ -\frac{1}{2} + \frac{\sin \sqrt{2}\lambda\pi}{4\sqrt{2}\lambda\pi} \right\} + \frac{1}{4} \left(\frac{-\lambda^2}{8} \frac{\lambda^2}{1+\delta\lambda^2} \right)^2 \right]
\end{aligned}$$

$$H_1(\lambda) = \frac{105}{8(12\lambda)^2} + \frac{\lambda^4}{4} \left(\frac{1}{32}\right)^2 + \frac{\lambda^4}{8} \left(\frac{1}{16}\right)^2 + \frac{\lambda^4}{8} \left(\frac{1-\lambda^2}{1+\lambda^2}\right)^2 \left(\frac{1}{32}\right)^2 + \frac{\lambda^2}{4} \left(\frac{1-\lambda^2}{1+\lambda^2}\right)^2$$

$$= \frac{105}{8(12\lambda)^2} + \frac{3\lambda^4}{(16)^3} + \frac{\lambda^4}{2(16)^3} = \frac{105}{(12\lambda)^3} + \frac{7\lambda^4}{2(16)^3}$$

$$H_2(\lambda) = + \frac{35}{288\pi} + \frac{5\lambda^4}{36\pi(1+\lambda^2)} - \frac{\lambda^6}{\pi(1+\lambda^2)(1+\lambda^2)^2}$$

$$= \frac{1}{\pi} \left[\frac{35}{288} + \frac{\lambda^4}{(1+\lambda^2)} \left\{ \frac{5}{36} - \frac{\lambda^2}{(1+\lambda^2)(1+\lambda^2)} \right\} \right]$$

$$H_3(\lambda) = \left(\frac{\pi^2}{32} + \frac{5}{16} \right) + 2\lambda^4 \frac{(1-\lambda^2)^2}{(1+\lambda^2)^4} + \frac{2\lambda^4(1-\lambda^2)}{(1+\lambda^2)^3} - \frac{128\lambda^6(1-\lambda^2)}{(1+\lambda^2)^5} g + \frac{\lambda^4}{(1+\lambda^2)^2} \left(\frac{\pi^2}{6} + 3 \right)$$

$$- \frac{128\lambda^6}{(1+\lambda^2)^4} - \frac{128\lambda^6(1-\lambda^2)}{(1+\lambda^2)^5} g + \frac{128\lambda^6}{(1+\lambda^2)^4} g + \frac{64\lambda^6}{(1+\lambda^2)^4} - \frac{32\lambda^6}{(1+\lambda^2)^3}$$

$$- \frac{128 \times 16 \lambda^6}{(1+\lambda^2)^5} g + \frac{8\lambda^6}{(1+\lambda^2)^2} \left(\frac{\pi^2}{6} - 1 \right) + \frac{128\lambda^6}{(1+\lambda^2)^4} - \frac{128 \times 16 \lambda^6}{(1+\lambda^2)^5} g$$

$$= \left(\frac{\pi^2}{32} + \frac{3}{16} \right) + \frac{64\lambda^6}{(1+\lambda^2)^4} g + \frac{\lambda^4}{1+\lambda^2} \frac{\pi^2}{6} + \frac{\lambda^4(1-\lambda^2)}{(1+\lambda^2)^2} + \frac{2\lambda^4}{(1+\lambda^2)^2} + \frac{2\lambda^4(1-24\lambda^2)}{(1+\lambda^2)^3}$$

$$g = \frac{\cos \sqrt{2}\pi\lambda}{(\sqrt{2}\pi\lambda \sinh \sqrt{2}\pi\lambda)}$$

$$\begin{aligned} \mathcal{E}_2 &= \frac{1}{24} \left(\frac{f}{R} \right)^2 \left\{ \left(\frac{f\pi^2}{16} \right)^2 \frac{3}{4} + \left(\frac{f\pi^2}{4} \right)^2 \lambda^4 \frac{1}{4} + 2 \left(\frac{f\pi^2}{8} \right)^2 \lambda^2 \frac{1}{4} \right\} \\ &= \frac{\left(\frac{f}{R} \right)^2 \left(\frac{f}{4} \right)^2 \pi^4 \left\{ \frac{1}{512} + \frac{\lambda^4}{96} + \frac{\lambda^2}{192} \right\}}{\end{aligned}$$

$$\begin{aligned} \text{Total energy} &= \left(\frac{Q}{R} \right)^4 \left(\frac{f}{4} \right)^2 \left\{ H_1(\lambda) (f\pi^2)^2 + H_2(\lambda) (f\pi^2) + H_3(\lambda) \right\} \\ &+ \left(\frac{f}{R} \right)^2 \left(\frac{f}{4} \right)^2 \pi^4 \left\{ \frac{1}{512} + \frac{\lambda^2}{192} + \frac{\lambda^4}{96} \right\} - \left(\frac{f}{E} \right) \left(\frac{Q}{R} \right)^2 \left(\frac{f}{4} \right)^2 \left(\frac{\pi^2 \lambda^2}{8} \right) - \frac{1}{2} \left(\frac{f}{E} \right) \end{aligned}$$

Thus

$$\begin{aligned} \left(\frac{f}{E} \right) \left(\frac{Q}{R} \right)^2 \frac{\pi^2 \lambda^2}{4} &= \left(\frac{Q}{R} \right)^4 \left\{ 4 H_1 (f\pi^2)^2 + 3 H_2 (f\pi^2) + 2 H_3 \right\} \\ &+ \left(\frac{f}{R} \right)^2 \pi^4 \left\{ \frac{1}{256} + \frac{\lambda^2}{96} + \frac{\lambda^4}{48} \right\} \end{aligned}$$

$$\lambda^2 K = f^2 \left\{ 16 H_1 (f\pi^2)^2 + 12 H_2 f + \frac{8 H_3}{\pi^2} \right\} + \frac{1}{f} \pi^2 \left\{ \frac{1}{34} + \frac{\lambda^2}{24} + \frac{\lambda^4}{12} \right\}$$

$$= \frac{\pi^2}{f^2} \left\{ 64 H_1 \left(\frac{f}{E} \right)^2 + \left[\frac{1}{64} + \frac{\lambda^2}{24} + \frac{\lambda^4}{12} \right] \right\} + \frac{8 H_3}{\pi^2} \left(\frac{f}{E} \right)^2 + 24 H_2 \left(\frac{f}{E} \right)$$

$$= 2 \left\{ 512 H_1 H_3 \left(\frac{f}{E} \right)^2 + H_3 \left[\frac{1}{8} + \frac{\lambda^2}{3} + \frac{2\lambda^4}{3} \right] \right\}^{\frac{1}{2}} + 24 H_2 \left(\frac{f}{E} \right)$$

$$f_{\min}^2 = \pi^2 \left\{ \frac{8 H_1}{H_3} \left(\frac{f}{E} \right)^2 + \frac{1}{H_3} \left[\frac{1}{512} + \frac{\lambda^2}{192} + \frac{\lambda^4}{96} \right] \right\}^{\frac{1}{2}}$$

$$\left(\frac{Q}{R} \right)^2 = \left(\frac{f}{R} \right) \pi^2 \left\{ \frac{8 H_1}{H_3} \left(\frac{f}{E} \right)^2 + \frac{1}{H_3} \left[\frac{1}{512} + \frac{\lambda^2}{192} + \frac{\lambda^4}{96} \right] \right\}^{\frac{1}{2}}$$

$$\begin{aligned}
\frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} &= \frac{E}{R^2} \frac{f \pi^2 \lambda^2}{4} \left\{ \frac{f \pi^2}{64} \left[(1 - \cos \frac{\pi x}{a}) (1 - \cos \frac{2\pi y}{b}) \right. \right. \\
&\quad \left. \left. - (1 + \cos \frac{\pi x}{a}) (1 + 2 \cos \frac{\pi y}{b} + \cos \frac{2\pi y}{b}) \right] \right. \\
&\quad \left. + \cos \frac{\pi x}{2a} \cos \frac{\pi y}{b} \right\} \\
&= \frac{E}{R^2} \frac{f \pi^2 \lambda^2}{4} \left\{ \frac{f \pi^2}{64} (-2 \cos \frac{\pi x}{a} - 2 \cos \frac{2\pi y}{b} - 2 \cos \frac{\pi y}{b} - 2 \cos \frac{\pi x}{a} \cos \frac{\pi y}{b}) \right. \\
&\quad \left. + \cos \frac{\pi x}{2a} \cos \frac{\pi y}{b} \right\} \\
&= \frac{E}{R^2} \frac{f \pi^2 \lambda^2}{4} \left\{ \cos \frac{\pi x}{2a} \cos \frac{\pi y}{b} - \frac{f \pi^2}{32} (\cos \frac{\pi x}{a} + \cos \frac{\pi y}{b} + \cos \frac{2\pi y}{b} + \cos \frac{\pi x}{a} \cos \frac{\pi y}{b}) \right\}
\end{aligned}$$

The particular integral

$$\begin{aligned}
F &= \frac{E}{R^2} \frac{f \pi^2 \lambda^2}{4} \left\{ \frac{\cos \frac{\pi x}{2a} \cos \frac{\pi y}{b}}{\left\{ \left(\frac{\pi}{2a} \right)^2 + \left(\frac{\pi}{b} \right)^2 \right\}^2} - \frac{f \pi^2}{32} \left[\frac{\cos \frac{\pi x}{a}}{\left(\frac{\pi}{a} \right)^4} + \frac{\cos \frac{\pi y}{b}}{\left(\frac{\pi}{b} \right)^4} + \frac{\cos \frac{2\pi y}{b}}{\left(\frac{2\pi}{b} \right)^4} \right. \right. \\
&\quad \left. \left. + \frac{\cos \frac{\pi x}{a} \cos \frac{\pi y}{b}}{\left\{ \left(\frac{\pi}{a} \right)^2 + \left(\frac{\pi}{b} \right)^2 \right\}^2} \right] \right\} \\
&= E \left(\frac{a}{R} \right)^2 \frac{f \lambda^2}{4} \left\{ \frac{\cos \frac{\pi x}{2a} \cos \frac{\pi y}{b}}{\left(\frac{\pi}{b} \right)^2 \left[\frac{1}{4} + \lambda^2 \right]^2} - \frac{f \pi^2}{32} \left[\frac{\cos \frac{\pi x}{a}}{\left(\frac{\pi}{a} \right)^2} + \frac{\cos \frac{\pi y}{b}}{\left(\frac{\pi}{b} \right)^2 \lambda^4} \right. \right. \\
&\quad \left. \left. + \frac{\cos \frac{2\pi y}{b}}{\left(\frac{\pi}{a} \right)^2 16 \lambda^4} + \frac{\cos \frac{\pi x}{a} \cos \frac{\pi y}{b}}{\left(\frac{\pi}{a} \right)^2 [1 + \lambda^2]^2} \right] \right\}
\end{aligned}$$

Write the complete solution as

$$F = E \left(\frac{q}{R} \right)^2 \frac{\lambda^2}{4} \frac{1}{(\frac{\pi}{a})^2} \left[\frac{16 \cos \frac{\pi}{2a} \cos \frac{\pi \lambda}{4}}{(1+4\lambda^2)^2} - \frac{\lambda^2}{32} \left\{ \cos \frac{\pi \lambda}{a} + \frac{1}{\lambda^2} \cos \frac{\pi \lambda}{b} + \frac{1}{4\lambda^2} \cos \frac{2\pi \lambda}{b} \right. \right. \\ \left. \left. + \frac{\cos \frac{\pi \lambda}{a} \cos \frac{\pi \lambda}{b}}{(1+\lambda^2)^2} \right\} + a_0 \left(\frac{\pi \lambda}{a} \right)^2 + \left\{ a_1 \cosh \frac{\pi \lambda x}{a} + b_1 \left(\frac{\pi \lambda x}{a} \right) \sinh \frac{\pi \lambda x}{a} \right\} \cos \frac{\pi \lambda}{b} \\ \left. + \left\{ a_2 \cosh \frac{2\pi \lambda x}{a} + b_2 \left(\frac{2\pi \lambda x}{a} \right) \sinh \frac{2\pi \lambda x}{a} \right\} \cos \frac{2\pi \lambda}{b} \right]$$

$$\hat{O}_\chi = \frac{\delta^2 F}{\delta q^2} = E \left(\frac{q}{R} \right)^2 \frac{\lambda^2}{4} \left[- \frac{16 \lambda^2 \cos \frac{\pi \lambda}{2a} \cos \frac{\pi \lambda}{4}}{(1+4\lambda^2)^2} + \frac{\lambda^2}{32} \left\{ \frac{1}{\lambda^2} \cos \frac{\pi \lambda}{b} + \frac{1}{4\lambda^2} \cos \frac{2\pi \lambda}{b} + \frac{\lambda^2 \cos \frac{\pi \lambda}{a} \cos \frac{\pi \lambda}{b}}{(1+\lambda^2)^2} \right\} \right]$$

$$- \lambda^2 \left\{ a_1 \cosh \frac{\pi \lambda x}{a} + b_1 \left(\frac{\pi \lambda x}{a} \right) \sinh \frac{\pi \lambda x}{a} \right\} \cos \frac{\pi \lambda}{b} \\ - 4\lambda^2 \left\{ a_2 \cosh \frac{2\pi \lambda x}{a} + b_2 \left(\frac{2\pi \lambda x}{a} \right) \sinh \frac{2\pi \lambda x}{a} \right\} \cos \frac{2\pi \lambda}{b} \right]$$

$$\hat{O}_\chi = E \left(\frac{q}{R} \right)^2 \frac{\lambda^2}{4} \left\{ \left[\frac{\lambda^2}{32} \left\{ \frac{1}{\lambda^2} - \frac{\lambda^2}{(1+\lambda^2)^2} \right\} - \lambda^2 \left\{ a_1 \cosh \pi \lambda + b_1 \pi \lambda \sinh \pi \lambda \right\} \right] \cos \frac{\pi \lambda}{b} \right. \\ \left. + \left[\frac{\lambda^2}{32} \left\{ \frac{1}{4\lambda^2} \right\} - 4\lambda^2 \left\{ a_2 \cosh 2\pi \lambda + b_2 2\pi \lambda \sinh 2\pi \lambda \right\} \right] \cos \frac{2\pi \lambda}{b} \right\}$$

thus

$$\begin{aligned} a_1 \cosh \pi \lambda + b_1 \pi \lambda \sinh \pi \lambda &= \frac{f \pi^2}{32} \left\{ \frac{1}{\lambda^2} - \frac{1}{(1+\lambda^2)^2} \right\} \\ a_2 \cosh \pi \lambda + b_2 \pi \lambda \sinh \pi \lambda &= \frac{f \pi^2}{32} \left\{ \frac{1}{16 \lambda^4} \right\} \end{aligned}$$

$$\tilde{G}_f^{\text{TF}} = E \left(\frac{q}{R} \right)^2 \frac{f \lambda^2}{4} \left[- \frac{4 \cos \frac{\pi \lambda}{2a} \cos \frac{\pi \lambda}{b}}{(1+4\lambda^2)^2} + \frac{f \pi^2}{32} \left\{ \cos \frac{\pi \lambda}{a} + \frac{\cos \frac{\pi \lambda}{b} \cos \frac{\pi \lambda}{3}}{(1+\lambda^2)^2} \right\} + 2a_0 \right]$$

$$+ \lambda^2 \left\{ (a_1 + 2b_1) \cosh \frac{\pi \lambda}{a} + b_1 \left(\frac{\pi \lambda}{a} \right) \sinh \frac{\pi \lambda}{a} \right\} \cos \frac{\pi \lambda}{b} + 4\lambda^2 \left\{ (a_2 + 2b_2) \cosh \frac{2\pi \lambda}{a} + b_2 \left(\frac{2\pi \lambda}{a} \right) \sinh \frac{2\pi \lambda}{a} \right\} \cos \frac{2\pi \lambda}{b} \right]$$

$$\frac{1}{2} \left(\frac{\partial \tilde{U}}{\partial q} \right)^2 = \left(\frac{q}{R} \right)^2 \frac{f \lambda^2}{4} \left[\frac{f \pi^2}{32} + \frac{f \pi^2}{32} \cos \frac{\pi \lambda}{a} - \frac{f \pi^2}{32} \cos \frac{2\pi \lambda}{b} - \frac{f \pi^2}{32} \cos \frac{\pi \lambda}{a} \cos \frac{2\pi \lambda}{b} \right]$$

$$\frac{\partial \tilde{U}}{\partial q} = \left(\frac{q}{R} \right)^2 \frac{f \lambda^2}{4} \left[- \frac{4 \cos \frac{\pi \lambda}{2a} \cos \frac{\pi \lambda}{b}}{(1+4\lambda^2)^2} + \frac{f \pi^2}{32} \left\{ -1 + \cos \frac{2\pi \lambda}{b} + \cos \frac{\pi \lambda}{a} \cos \frac{2\pi \lambda}{b} + \frac{\cos \frac{\pi \lambda}{a} \cos \frac{\pi \lambda}{3}}{(1+\lambda^2)^2} \right\} \right]$$

$$+ 2a_0 + \lambda^2 \left\{ (a_1 + 2b_1) \cosh \frac{\pi \lambda}{a} + b_1 \left(\frac{\pi \lambda}{a} \right) \sinh \frac{\pi \lambda}{a} \right\} \cos \frac{\pi \lambda}{b} + 4\lambda^2 \left\{ (a_2 + 2b_2) \cosh \frac{2\pi \lambda}{a} + b_2 \left(\frac{2\pi \lambda}{a} \right) \sinh \frac{2\pi \lambda}{a} \right\} \cos \frac{2\pi \lambda}{b} \right]$$

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$$\frac{1}{R} = \left(\frac{a}{R}\right)^3 \frac{f\lambda^2}{4} \left[-\frac{1}{\pi\lambda} \frac{4\cos\frac{\pi\lambda}{2a} \sin\frac{\pi\lambda}{b}}{(1+4\lambda^2)^2} + \left(-\frac{f\lambda^2}{32} + 2a_0'\right) \frac{1}{\lambda} \left(\frac{1}{b}\right) + \dots \right]$$

$$\left(\frac{q}{b}\right)_{\text{at } q=b} = \left(\frac{q}{R}\right)^2 \frac{f\lambda^2}{4} \left[-\frac{f\lambda^2}{32} + 2a_0 \right]$$

$$\begin{aligned} \sigma_y \text{ at } y=b &= E \left(\frac{q}{R}\right)^2 \frac{f\lambda^2}{4} \left[+ \frac{4\cos\frac{\pi\lambda}{2a}}{(1+4\lambda^2)^2} + \frac{f\lambda^2}{32} \left\{ \cos\frac{\pi\lambda}{a} - \frac{\cos\frac{\pi\lambda}{b}}{(1+\lambda^2)^2} \right\} + 2a_0 \right. \\ &\quad \left. - \lambda^2 \left\{ (a_1 + 2b_1) \cos\frac{\pi\lambda}{a} + b_1 \left(\frac{\pi\lambda}{a}\right) \sin\frac{\pi\lambda}{a} \right\} \right. \\ &\quad \left. + 4\lambda^2 \left\{ (a_2 + 2b_2) \cos\frac{2\pi\lambda}{a} + b_2 \left(\frac{2\pi\lambda}{a}\right) \sin\frac{2\pi\lambda}{a} \right\} \right] \end{aligned}$$

$$\begin{aligned} -\frac{\sigma}{E} &= \left(\frac{q}{R}\right)^2 \frac{f\lambda^2}{4} \left[\frac{f}{\pi} \frac{1}{(1+4\lambda^2)^2} - \frac{1}{\pi} \left\{ (a_1 + b_1) \sin\frac{\pi\lambda}{a} + b_1 \pi\lambda \cos\frac{\pi\lambda}{a} \right\} \right. \\ &\quad \left. + 2a_0 + \frac{2\lambda}{\pi} \left\{ (a_1 + b_1) \sin\frac{2\pi\lambda}{a} + b_2 2\pi\lambda \cos\frac{2\pi\lambda}{a} \right\} \right] \end{aligned}$$

$$T_{xy} = E \left(\frac{g^2}{R} \right)^2 \frac{h^2}{4} \left[- \frac{8 h \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}}{(1+h^2)^2} + \frac{4 h^2}{32} \left\{ \frac{\lambda \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}}{(1+\lambda^2)^2} \right\} \right]$$

$$+ \lambda^2 \left\{ (a_1 + b_1) \sinh \frac{\pi \lambda y}{a} + b_1 \left(\frac{\pi \lambda y}{a} \right) \cosh \frac{\pi \lambda y}{a} \right\} \sin \frac{\pi x}{b}$$

$$+ 4 h^2 \left\{ (a_2 + b_2) \sinh \frac{2 \pi \lambda y}{a} + b_2 \left(\frac{2 \pi \lambda y}{a} \right) \cosh \frac{2 \pi \lambda y}{a} \right\} \sin \frac{2 \pi x}{b} \Big]$$

$$\therefore$$

$$(a_2 + b_2) \sinh 2\pi\lambda + b_2 (2\pi\lambda) \cosh 2\pi\lambda = 0$$

$$(a_1 + b_1) \sinh \pi\lambda + b_1 (\pi\lambda) \cosh \pi\lambda = \frac{1}{\lambda} \frac{8}{(1+4\lambda^2)^2}$$

$$\therefore -\frac{\sigma}{E} = \left(\frac{g^2}{R} \right)^2 \frac{h^2}{4} (2a_0), \quad \left(\frac{\pi y}{b} \right)_{at y=b} = -\frac{\sigma}{E} - \left(\frac{g^2}{R} \right)^2 \frac{h^2}{4} \frac{h^2}{32}$$

$$\frac{\Delta p}{E a_0 b} = - \left(\frac{g}{E} \right)^2 - \left(\frac{g}{E} \right)^2 \left(\frac{g}{R} \right)^2 \left(\frac{\pi \lambda^2}{8} \right)$$

$$\begin{aligned}
 \psi = E(R)^2 \frac{1}{4} \int & \left[\left(\frac{\pi^2}{32} - \frac{16\lambda^4}{(1+4\lambda^2)^2} \cos \frac{\pi x}{2a} + \frac{\pi^2 \lambda^4}{32(1+4\lambda^2)^2} \cos \frac{\pi x}{a} - \lambda^4 \left\{ a, \cosh \frac{\pi x}{a} + b, \left(\frac{\pi x}{a} \right) \sinh \frac{\pi x}{a} \right\} \cos \frac{\pi x}{8} \right. \right. \\
 & \left. \left. + \left(\frac{\pi^2}{128} - 4\lambda^4 \left\{ a_2 \cosh \frac{2\pi x}{a} + b, \left(\frac{2\pi x}{a} \right) \sinh \frac{2\pi x}{a} \right\} \right) \cos \frac{2\pi x}{8} \right]
 \end{aligned}$$

$$\frac{1}{g_{ab}} \int_0^a \int_0^b \left(\frac{\psi}{R} \right)^2 dx dy = \left(\frac{a}{R} \right)^4 \left(\frac{1}{4} \right)^2 \left[\frac{1}{4} \left(\frac{\pi^2}{32} \right)^2 + \frac{1}{8} \left(\frac{16\lambda^4}{(1+4\lambda^2)^2} \right)^2 + \frac{1}{8} \left\{ \frac{\pi^2 \lambda^4}{32(1+4\lambda^2)^2} \right\}^2 \right]$$

$$\begin{aligned}
 & - \frac{1}{2} \frac{\pi^2 \lambda^4}{32} \int_0^1 \left\{ a, \cosh \frac{\pi x}{a} + b, \left(\frac{\pi x}{a} \right) \sinh \frac{\pi x}{a} \right\} d\left(\frac{x}{a}\right) \\
 & + \frac{8\lambda^8}{(1+4\lambda^2)^2} \int_0^1 \cos \frac{\pi x}{2a} \left\{ a, \cosh \frac{\pi x}{a} + b, \left(\frac{\pi x}{a} \right) \sinh \frac{\pi x}{a} \right\} d\left(\frac{x}{a}\right) \\
 & - \frac{1}{2} \frac{\pi^2 \lambda^4}{32(1+4\lambda^2)^2} \int_0^1 \cos \frac{\pi x}{a} \left\{ a, \cosh \frac{\pi x}{a} + b, \left(\frac{\pi x}{a} \right) \sinh \frac{\pi x}{a} \right\} d\left(\frac{x}{a}\right) \\
 & + \frac{1}{4} \lambda^8 \int_0^1 \left\{ a, \cosh \frac{\pi x}{a} + b, \left(\frac{\pi x}{a} \right) \sinh \frac{\pi x}{a} \right\}^2 d\left(\frac{x}{a}\right) + \frac{1}{4} \left(\frac{\pi^2 \lambda^4}{128} \right)^2 \\
 & - \frac{\pi^2 \lambda^4}{64} \int_0^1 \left\{ a_2 \cosh \frac{2\pi x}{a} + b, \left(\frac{2\pi x}{a} \right) \sinh \frac{2\pi x}{a} \right\} d\left(\frac{x}{a}\right) \\
 & + 4\lambda^8 \int_0^1 \left\{ a_2 \cosh \frac{2\pi x}{a} + b, \left(\frac{2\pi x}{a} \right) \sinh \frac{2\pi x}{a} \right\}^2 d\left(\frac{x}{a}\right)
 \end{aligned}$$

$$\begin{aligned}
 \bar{y} = E\left(\frac{\rho}{R}\right)\left(\frac{f}{4}\right) & \left[\frac{f^2 \lambda^2}{32} \cos \frac{\pi x}{a} \right. \\
 & + \left\{ -\frac{4\lambda^2}{(1+4\lambda^2)^2} \cos \frac{\pi x}{2a} + \frac{f^2 \lambda^2}{32(1+\lambda^2)^2} \cos \frac{\pi x}{a} + b_1 \left(\frac{\pi \lambda x}{a}\right) \sinh \frac{\pi \lambda x}{a} \right\} \cos \frac{2\pi x}{a} \\
 & + \left\{ 4\lambda^4 \left[(a_2 + 2b_2) \cos \frac{2\pi \lambda x}{a} + b_2 \left(\frac{2\pi \lambda x}{a}\right) \sinh \frac{2\pi \lambda x}{a} \right] \right\} \cos \frac{2\pi x}{a} \left. \right] - \frac{Q}{E}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{g_{ob}} \int_0^b \int_0^b \left(\frac{y}{E} \right)^2 dx dy &= \left(\frac{\rho}{R} \right)^2 \left(\frac{f}{4} \right)^2 \left[\frac{f}{4} \left(\frac{f^2 \lambda^2}{32} \lambda^2 \right)^2 + \frac{f}{8} \left(\frac{4\lambda^2}{(1+4\lambda^2)^2} \right)^2 + \frac{f}{8} \left\{ \frac{f^2 \lambda^2}{32(1+\lambda^2)^2} \right\}^2 \right. \\
 &- \frac{2\lambda^6}{(1+4\lambda^2)^2} \int_0^1 \cos \frac{\pi x}{2a} \left\{ (a_1 + 2b_1) \cos \frac{\pi \lambda x}{a} + b_1 \left(\frac{\pi \lambda x}{a}\right) \sinh \frac{\pi \lambda x}{a} \right\} d\left(\frac{x}{a}\right) \\
 &+ \frac{f}{2} \frac{f^2 \lambda^2}{32(1+\lambda^2)^2} \int_0^1 \cos \frac{\pi x}{a} \left\{ (a_1 + 2b_1) \cos \frac{\pi \lambda x}{a} + b_1 \left(\frac{\pi \lambda x}{a}\right) \sinh \frac{\pi \lambda x}{a} \right\} d\left(\frac{x}{a}\right) \\
 &+ \frac{\lambda^8}{4} \int_0^1 \int_0^1 \left\{ (a_1 + 2b_1) \cos \frac{\pi \lambda x}{a} + b_1 \left(\frac{\pi \lambda x}{a}\right) \sinh \frac{\pi \lambda x}{a} \right\}^2 d\left(\frac{x}{a}\right) \\
 &+ 4\lambda^8 \int_0^1 \left\{ (a_2 + 2b_2) \cos \frac{2\pi \lambda x}{a} + b_2 \left(\frac{2\pi \lambda x}{a}\right) \sinh \frac{2\pi \lambda x}{a} \right\}^2 d\left(\frac{x}{a}\right) \left. \right] + \frac{f^2 Q^2}{E^2}
 \end{aligned}$$

$$\begin{aligned} \varepsilon_q = E(\frac{q}{R}) \frac{1}{4} \int_0^1 \left[-\frac{f\lambda^3}{(1+4\lambda^2)^2} \sin \frac{\pi x}{2a} + \frac{f\lambda^2}{32} \frac{\lambda^3}{(1+\lambda)^2} \sin \frac{\pi x}{a} + \lambda^6 \left[(a_1+b_1) \sinh \frac{\pi \lambda x}{a} + b_1 \left(\frac{\pi \lambda x}{a} \right) \cosh \frac{\pi \lambda x}{a} \right] \sin \frac{\pi x}{2} \right. \\ \left. + 4\lambda^4 \left\{ (a_2+b_2) \sinh \frac{2\pi \lambda x}{a} + b_2 \left(\frac{2\pi \lambda x}{a} \right) \cosh \frac{2\pi \lambda x}{a} \right\} \sin \frac{2\pi x}{6} \right] \end{aligned}$$

$$\begin{aligned} \frac{1}{ab} \int_0^a \int_0^b \frac{\varepsilon_q^2}{(E)} dx dy = \left(\frac{q}{R} \right) \left(\frac{1}{4} \right)^2 \left[\frac{1}{4} \left[\frac{f\lambda^3}{(1+4\lambda)^2} \right]^2 + \frac{1}{4} \left[\frac{f\lambda^2}{32} \frac{\lambda^3}{(1+\lambda)^2} \right]^2 \right. \\ \left. - \frac{f\lambda^7}{(1+4\lambda)^2} \int_0^1 \sin \frac{\pi x}{2a} \left\{ (a_1+b_1) \sinh \frac{\pi \lambda x}{a} + b_1 \left(\frac{\pi \lambda x}{a} \right) \cosh \frac{\pi \lambda x}{a} \right\} d\left(\frac{x}{a}\right) \right. \\ \left. + \frac{f\lambda^2}{32} \frac{\lambda^3}{(1+\lambda)^2} \int_0^1 \sin \frac{\pi x}{a} \left\{ (a_1+b_1) \sinh \frac{\pi \lambda x}{a} + b_1 \left(\frac{\pi \lambda x}{a} \right) \cosh \frac{\pi \lambda x}{a} \right\} d\left(\frac{x}{a}\right) \right. \\ \left. + \frac{1}{2} \lambda^6 \int_0^1 \left\{ (a_1+b_1) \sinh \frac{\pi \lambda x}{a} + b_1 \left(\frac{\pi \lambda x}{a} \right) \cosh \frac{\pi \lambda x}{a} \right\}^2 d\left(\frac{x}{a}\right) \right. \\ \left. + f\lambda^8 \int_0^1 \left\{ (a_2+b_2) \sinh \frac{2\pi \lambda x}{a} + b_2 \left(\frac{2\pi \lambda x}{a} \right) \cosh \frac{2\pi \lambda x}{a} \right\}^2 d\left(\frac{x}{a}\right) \right] \end{aligned}$$

$$\int_0^1 \operatorname{erfc} \frac{\pi \lambda x}{a} d\left(\frac{x}{a}\right) = \frac{1}{\pi \lambda} \operatorname{erfc} \pi \lambda$$

$$\int_0^1 \frac{\pi \lambda x}{a} \operatorname{erfc} \frac{\pi \lambda x}{a} d\left(\frac{x}{a}\right) = \frac{1}{\pi \lambda} \int_0^{\pi \lambda} \theta \operatorname{erfc} \theta d\theta = \frac{1}{\pi \lambda} \left[\pi \lambda \operatorname{erfc} \pi \lambda - \operatorname{erfc} \pi \lambda \right] = \operatorname{erfc} \pi \lambda - \frac{\operatorname{erfc} \pi \lambda}{\pi \lambda}$$

$$\int_0^1 \operatorname{erfc} \frac{\pi x}{2a} \operatorname{erfc} \frac{\pi \lambda x}{a} d\left(\frac{x}{a}\right) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left[\operatorname{erfc}(\lambda \lambda + i)\theta + \operatorname{erfc}(\lambda \lambda - i)\theta \right] \theta d\theta$$

$$= \frac{1}{\pi} \left[\frac{\operatorname{erfc}(\lambda \lambda + i)^{\frac{\pi}{2}}}{\lambda \lambda + i} + \frac{\operatorname{erfc}(\lambda \lambda - i)^{\frac{\pi}{2}}}{\lambda \lambda - i} \right] = \frac{2}{\pi} \frac{\operatorname{erfc} \pi \lambda}{(1 + 4\lambda^2)}$$

$$\int_0^1 \operatorname{erfc} \frac{\pi x}{2a} \left(\frac{\pi \lambda x}{a} \operatorname{erfc} \frac{\pi \lambda x}{a} d\left(\frac{x}{a}\right) \right) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \lambda \theta \left[\operatorname{erfc}(\lambda \lambda + i)\theta + \operatorname{erfc}(\lambda \lambda - i)\theta \right] d\theta$$

$$= \frac{\lambda}{\pi} \int_0^{\frac{\pi}{2}} \left[\frac{\operatorname{erfc}(\lambda \lambda + i)^{\frac{\pi}{2}}}{\lambda \lambda + i} + \frac{\operatorname{erfc}(\lambda \lambda - i)^{\frac{\pi}{2}}}{\lambda \lambda - i} \right] \frac{\pi}{2} - \left[\frac{\operatorname{erfc}(\lambda \lambda + i)^{\frac{\pi}{2}}}{(\lambda \lambda + i)^2} + \frac{\operatorname{erfc}(\lambda \lambda - i)^{\frac{\pi}{2}}}{(\lambda \lambda - i)^2} \right]$$

$$= \frac{\lambda}{\pi} \left[\frac{\pi \operatorname{erfc} \pi \lambda}{1 + 4\lambda^2} - \frac{\lambda \operatorname{erfc} \pi \lambda}{(1 + 4\lambda^2)^2} \right] = \left[\frac{\lambda \operatorname{erfc} \pi \lambda}{(1 + 4\lambda^2)} - \frac{16\lambda^2 \operatorname{erfc} \pi \lambda}{\pi (1 + 4\lambda^2)^2} \right]$$

$$\int_0^1 \operatorname{erfc} \frac{\pi x}{a} \operatorname{erfc} \frac{\pi \lambda x}{a} d\left(\frac{x}{a}\right) = \frac{1}{\pi} \int_0^{\pi} \left[\operatorname{erfc}(\lambda + i)\theta + \operatorname{erfc}(\lambda - i)\theta \right] \theta d\theta$$

$$= \frac{1}{2\pi} \left[\frac{\operatorname{erfc}(\lambda + i)^{\frac{\pi}{2}}}{\lambda + i} + \frac{\operatorname{erfc}(\lambda - i)^{\frac{\pi}{2}}}{\lambda - i} \right] = -\frac{\lambda}{\pi} \frac{\operatorname{erfc} \lambda \pi}{1 + \lambda^2}$$

$$\begin{aligned}
 \int_0^1 \frac{co \frac{\pi x}{a} \left(\frac{\pi \lambda x}{a} \right) \sinh \frac{\pi \lambda x}{a} d\left(\frac{x}{a}\right)}{2\pi} &= \frac{1}{2\pi} \int_0^1 \lambda \theta \left[\sinh(\lambda+i)\theta + \sinh(\lambda-i)\theta \right] d\theta \\
 &= \frac{1}{2\pi} \left[\pi \left\{ \frac{\cosh(\lambda+i)\pi}{\lambda+i} + \frac{\cosh(\lambda-i)\pi}{\lambda-i} \right\} - \left\{ \frac{\sinh(\lambda+i)\pi}{(\lambda+i)^2} + \frac{\sinh(\lambda-i)\pi}{(\lambda-i)^2} \right\} \right] \\
 &= \frac{1}{2\pi} \left[-\frac{2\lambda\pi \cosh \pi}{1+\lambda^2} - \frac{2(1-\lambda^2) \sinh \lambda \pi}{(1+\lambda^2)^2} \right]
 \end{aligned}$$

$$\int_0^1 \frac{\cosh^2 \frac{\pi \lambda x}{a} d\left(\frac{x}{a}\right)}{2} = \frac{1}{2} \int_0^1 \left[1 + \cosh \frac{2\pi \lambda x}{a} \right] d\left(\frac{x}{a}\right) = \frac{1}{2} + \frac{\sinh 2\pi \lambda}{2\pi \lambda}$$

$$\begin{aligned}
 \int_0^1 \left(\frac{\pi \lambda x}{a} \right) \sinh \frac{\pi \lambda x}{a} \cosh \frac{\pi \lambda x}{a} d\left(\frac{x}{a}\right) &= -\frac{1}{2} \int_0^1 \left(\frac{\pi \lambda x}{a} \right) \sinh \frac{2\pi \lambda x}{a} d\left(\frac{x}{a}\right) = \frac{1}{8\pi \lambda} \left[2\pi \lambda \cosh 2\pi \lambda - \sinh 2\pi \lambda \right] \\
 &= \frac{1}{4} \left[\cosh 2\pi \lambda - \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right]
 \end{aligned}$$

$$\begin{aligned}
 \int_0^1 \frac{\left(\frac{\pi \lambda x}{a} \right)^2 \sinh^2 \frac{\pi \lambda x}{a} d\left(\frac{x}{a}\right)}{2} &= \frac{1}{2} \int_0^1 \left(\frac{\pi \lambda x}{a} \right)^2 \left[\cosh \frac{2\pi \lambda x}{a} - 1 \right] d\left(\frac{x}{a}\right) = \frac{1}{2} \left[-\frac{1}{3} \left(\frac{\pi \lambda}{a} \right)^2 \right. \\
 &\quad \left. + \frac{1}{8\pi \lambda} \left\{ (x^2+2) \sinh x - 2x \cosh x \right\} \right]_0^{2\pi \lambda} = \frac{1}{2} \left[-\frac{(\pi \lambda)^2}{3} + \frac{1}{8\pi \lambda} \left\{ (4\pi^2 \lambda^2 + 2) \sinh 2\pi \lambda - 4\pi \lambda \cosh 2\pi \lambda \right\} \right] \\
 &= -\frac{(\pi \lambda)^2}{6} + \frac{1}{8} \left[\frac{(4\pi^2 \lambda^2 + 2) \sinh 2\pi \lambda}{2\pi \lambda} - 2 \cosh 2\pi \lambda \right]
 \end{aligned}$$

$$\int_0^1 \sin \frac{\pi x}{2a} \coth \frac{\pi x}{a} d\left(\frac{x}{a}\right) = \frac{1}{\pi i} \int_0^{\frac{\pi}{2}} [\coth(2\lambda+i)\theta - \coth(2\lambda-i)\theta] d\theta$$

$$= \frac{1}{\pi i} \left[\frac{\sinh(2\lambda+i)\theta}{2\lambda+i} - \frac{\sinh(2\lambda-i)\theta}{2\lambda-i} \right] = \frac{4\lambda}{\pi} \frac{\coth \pi \lambda}{1+4\lambda^2}$$

$$\int_0^1 \sin \frac{\pi x}{2a} \coth \frac{\pi x}{a} d\left(\frac{x}{a}\right) = \frac{1}{\pi i} 2\lambda \int_0^{\frac{\pi}{2}} \theta [\sinh(2\lambda+i)\theta - \sinh(2\lambda-i)\theta] d\theta$$

$$= \frac{2\lambda}{\pi i} \left[\frac{\pi}{2} \left\{ \frac{\coth(2\lambda+i)\frac{\pi}{2}}{2\lambda+i} \right\} - \left\{ \frac{\sinh(2\lambda+i)\frac{\pi}{2}}{(2\lambda+i)^2} - \frac{\sinh(2\lambda-i)\frac{\pi}{2}}{(2\lambda-i)^2} \right\} \right]$$

$$= \frac{2\lambda}{\pi} \left[-\frac{2\pi\lambda \sinh \pi \lambda}{(1+4\lambda^2)} + \frac{2(1-4\lambda^2) \coth \pi \lambda}{(1+4\lambda^2)^2} \right]$$

$$\int_0^1 \sin \frac{\pi x}{2a} \sinh \frac{\pi x}{a} d\left(\frac{x}{a}\right) = \frac{1}{2\pi i} \int_0^{\pi} [\coth(\lambda+i)\theta - \coth(\lambda-i)\theta] d\theta$$

$$= \frac{1}{2\pi i} \left[\frac{\sinh(\lambda+i)\pi}{\lambda+i} - \frac{\sinh(\lambda-i)\pi}{\lambda-i} \right] = \frac{1}{\pi} \frac{\sinh \lambda \pi}{(1+\lambda^2)}$$

$$\int_0^1 \sin \frac{\pi x}{a} \coth \frac{\pi x}{a} d\left(\frac{x}{a}\right) = \frac{\lambda}{2\pi i} \int_0^{\pi} \theta [\sinh(\lambda+i)\theta - \sinh(\lambda-i)\theta] d\theta$$

$$= \frac{\lambda}{2\pi i} \left[\pi \left\{ \frac{\coth(\lambda+i)\pi}{\lambda+i} - \frac{\coth(\lambda-i)\pi}{\lambda-i} \right\} - \left\{ \frac{\sinh(\lambda+i)\pi}{(\lambda+i)^2} - \frac{\sinh(\lambda-i)\pi}{(\lambda-i)^2} \right\} \right]$$

$$= \frac{\lambda}{2\pi} \left[\frac{2\pi \coth \lambda \pi}{1+\lambda^2} - \frac{4\lambda \sinh \lambda \pi}{(1+\lambda^2)^2} \right]$$

$$\int_0^1 \sinh \frac{\pi \lambda x}{a} d\left(\frac{x}{a}\right) = \frac{1}{2} \int_0^1 \left[\cosh \frac{2\pi \lambda x}{a} - \frac{1}{2} \right] d\left(\frac{x}{a}\right) = -\frac{1}{2} + \frac{\sinh \frac{2\pi \lambda}{2\pi \lambda}}{2\pi \lambda}$$

$$\begin{aligned} \int_0^1 \left(\frac{\pi \lambda}{a}\right)^2 \cosh \frac{\pi \lambda x}{a} d\left(\frac{x}{a}\right) &= \frac{1}{2} \int_0^1 \left(\frac{\pi \lambda}{a}\right)^2 \left[1 + \cosh \frac{2\pi \lambda x}{a} \right] d\left(\frac{x}{a}\right) \\ &= \frac{1}{6} (\pi \lambda)^2 + \frac{1}{2} \left[(4\pi^2 \lambda^2 \cdot 2) \frac{\sinh \frac{2\pi \lambda}{2\pi \lambda}}{2\pi \lambda} - 2 \cosh \frac{2\pi \lambda}{2\pi \lambda} \right] \end{aligned}$$

$$\begin{aligned}
\frac{1}{g\nu} \int_0^{\nu} \int_0^{\nu} \left(\frac{\delta \psi^2}{E} \right) dx dy &= \left(\frac{R}{K} \right)^4 \left(\frac{1}{4} \right)^2 \left[\frac{1}{4} \left(\frac{\pi^2}{32} \right)^2 + \frac{1}{8} \frac{2\pi^2 \lambda^8}{(1+\lambda^2)^4} + \frac{1}{8} \frac{\lambda^8}{(1+\lambda^2)^4} \left(\frac{\pi^2}{32} \right)^2 \right. \\
&\quad - \frac{1}{2} \frac{\pi^2}{32} \lambda^4 \left[(a_1 - b_1) \frac{\sin 6\pi\lambda}{\pi\lambda} + b_1 \cos 2\pi\lambda \right] \\
&\quad + \frac{8\lambda^8}{(1+\lambda^2)^2} \left[\frac{2}{\pi} \frac{\cos 6\pi\lambda}{(1+\lambda^2)} a_1 + \left\{ \frac{2\lambda \sin 6\pi\lambda}{(1+\lambda^2)} - \frac{16\lambda^2 \cos 6\pi\lambda}{\pi(1+\lambda^2)^2} \right\} b_1 \right] \\
&\quad + \frac{\pi^2}{64} \frac{\lambda^8}{(1+\lambda^2)^2} \left[+ \frac{\lambda}{\pi} \frac{\sin 6\pi\lambda}{(1+\lambda^2)} a_1 + \left\{ \frac{\lambda^2 \cos 6\pi\lambda}{(1+\lambda^2)} + \frac{\lambda(1-\lambda^2) \sin 6\pi\lambda}{\pi(1+\lambda^2)^2} \right\} b_1 \right] \\
&\quad + \frac{\lambda^8}{4} \left[a_1^2 \left\{ \frac{1}{2} + \frac{\sin 6\pi\lambda}{2\pi\lambda} \right\} + g a_1 a_1 \frac{1}{4} \left\{ \cos 6\pi\lambda - \frac{\sin 6\pi\lambda}{2\pi\lambda} + b_1^2 \left\{ -\frac{\pi\lambda^2}{6} + \frac{1}{8} \left[(4\pi\lambda^2 + 2) \frac{\sin 6\pi\lambda}{2\pi\lambda} - 2 \cos 6\pi\lambda \right] \right\} \right. \right. \\
&\quad \left. \left. + \frac{1}{4} \left(\frac{\pi^2}{128} \right)^2 \right] \right. \\
&\quad - \frac{\pi^2}{64} \lambda^4 \left[(a_2 - b_2) \frac{\sin 6\pi\lambda}{2\pi\lambda} + b_2 \cos 2\pi\lambda \right] \\
&\quad \left. + 4\lambda^8 \left[a_2^2 \left\{ \frac{1}{2} + \frac{\sin 6\pi\lambda}{4\pi\lambda} \right\} + g a_2 a_2 \frac{1}{4} \left\{ \cos 6\pi\lambda - \frac{\sin 6\pi\lambda}{4\pi\lambda} + b_2^2 \left\{ -\frac{4\pi^2 \lambda^2}{6} + \frac{1}{8} \left[(16\pi^2 \lambda^2 + 2) \frac{\sin 6\pi\lambda}{4\pi\lambda} - 2 \cos 6\pi\lambda \right] \right\} \right. \right. \right. \right.
\end{aligned}$$

$$\frac{1}{8ab} \int_0^a \int_0^b \left(\frac{\partial^2}{\partial x^2} \right)^2 dx dy = \left(\frac{a}{R} \right)^4 \left(\frac{b}{L} \right)^2 \left[\frac{\lambda^4}{4} \left(\frac{b^2}{32} \right)^2 + \frac{1}{8} \frac{16\lambda^4}{(1+4\lambda^2)^4} + \frac{1}{8} \frac{\lambda^4}{(1+\lambda^2)^4} \left(\frac{b^2}{32} \right)^2 \right]$$

$$- \frac{2\lambda^6}{(1+4\lambda^2)^2} \left\{ \frac{2}{\pi} \frac{\cosh \pi \lambda}{(1+4\lambda^2)} (a_1 + 2b_1) + \left[\frac{2\lambda \sinh \pi \lambda}{(1+4\lambda^2)} - \frac{16\lambda^2 \cosh \pi \lambda}{\pi (1+4\lambda^2)^2} \right] b_1 \right\}$$

$$- \frac{\pi^2}{64} \frac{\lambda^6}{(1+\lambda^2)^2} \left\{ \frac{\lambda}{\pi} \frac{\sinh \lambda \pi}{(1+\lambda^2)} (a_1 + 2b_1) + \left[\frac{\lambda^2 \cosh \lambda \pi}{(1+\lambda^2)} + \frac{\lambda(1-\lambda^2) \sinh \lambda \pi}{\pi (1+\lambda^2)^2} \right] b_1 \right\}$$

$$+ \frac{\lambda^8}{4} \left\{ (a_1 + 2b_1)^2 \left[\frac{1}{2} + \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right] + 2b_1 (a_1 + 2b_1) \frac{1}{4} \left[\cosh 2\pi \lambda - \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right] \right. \\ \left. + b_1^2 \left[-\frac{\pi \lambda^2}{6} + \frac{1}{8} [(4\pi^2 \lambda^2 + 2) \frac{\sinh 2\pi \lambda}{2\pi \lambda} - 2 \cosh 2\pi \lambda] \right] \right\}$$

$$+ 4\lambda^8 \left\{ (a_2 + 2b_2)^2 \left[\frac{1}{2} + \frac{\sinh 4\pi \lambda}{4\pi \lambda} \right] + 2b_2 (a_2 + 2b_2) \frac{1}{4} \left[\cosh 4\pi \lambda - \frac{\sinh 4\pi \lambda}{4\pi \lambda} \right] \right.$$

$$\left. + b_2^2 \left[-\frac{4\pi \lambda^2}{6} + \frac{1}{8} [(16\pi^2 \lambda^2 + 9) \frac{\sinh 4\pi \lambda}{4\pi \lambda} - 2 \cosh 4\pi \lambda] \right] \right\} + \frac{\lambda^6}{2} \left(\frac{b}{L} \right)^2$$

$$\begin{aligned}
\frac{1}{ab} \int_0^a \int_0^b \left(\frac{E_{xy}}{E} \right)^2 dx dy &= \left(\frac{a}{b} \right)^4 \left(\frac{a}{b} \right)^2 \left[\frac{1}{4} \frac{6^4 \lambda^6}{(1+4\lambda^2)^4} + \frac{1}{4} \frac{\lambda^6}{(1+\lambda^2)^4} \left(\frac{a^2}{32} \right)^2 \right. \\
&- \frac{8\lambda^2}{(1+4\lambda^2)^2} \left\{ \frac{4\lambda}{\pi} \frac{\cosh \pi \lambda}{1+4\lambda^2} (a_1+b_1) + \left[\frac{4\lambda^2 \sinh \pi \lambda}{(1+4\lambda^2)} + \frac{4\lambda(1-4\lambda^2) \cosh \pi \lambda}{\pi(1+4\lambda^2)^2} \right] b_1 \right\} \\
&+ \frac{1}{32} \frac{\pi^2 \lambda^2}{(1+\lambda^2)^2} \left\{ \frac{1}{\pi} \frac{\sinh \lambda \pi}{(1+\lambda^2)} (a_1+b_1) + \left[\frac{\lambda \cosh \lambda \pi}{(1+\lambda^2)} - \frac{2\lambda^2 \sinh \lambda \pi}{\pi(1+\lambda^2)^2} \right] b_1 \right\} \\
&+ \frac{\lambda^4}{2} \left\{ (a_1+b_1)^2 \left[-\frac{1}{2} + \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right] + 2b_1(a_1+b_1) \frac{1}{4} \left[\cosh 2\pi \lambda - \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right] \right. \\
&\quad \left. + b_1^2 \left[\frac{\pi^2 \lambda^2}{6} + \frac{1}{8} \left[(4\pi^2 \lambda^2 + 2) \frac{\sinh 2\pi \lambda}{2\pi \lambda} - 2 \cosh 2\pi \lambda \right] \right] \right\} \\
&+ 8\lambda^4 \left\{ (a_2+i_2)^2 \left[-\frac{1}{2} + \frac{\sinh 4\pi \lambda}{4\pi \lambda} \right] + 2b_2(a_2+i_2) \frac{1}{4} \left[\cosh 4\pi \lambda - \frac{\sinh 4\pi \lambda}{4\pi \lambda} \right] \right. \\
&\quad \left. + b_2^2 \left[\frac{4\pi^2 \lambda^2}{6} + \frac{1}{8} \left[(16\pi^2 \lambda^2 + 2) \frac{\sinh 4\pi \lambda}{4\pi \lambda} - 2 \cosh 4\pi \lambda \right] \right] \right\}
\end{aligned}$$

$$\cosh \pi \lambda \cdot a_1 + \pi \lambda \sinh \pi \lambda \cdot b_1 = \frac{f\pi^2}{32} \left\{ \frac{1}{\lambda^4} - \frac{1}{(1+\lambda^2)^2} \right\}$$

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$$\sinh \pi \lambda \cdot a_1 + (\sinh \pi \lambda + \pi \lambda \cosh \pi \lambda) b_1 = \frac{1}{\lambda} \frac{\rho}{(1+4\lambda^2)^2}$$

$$(\pi \lambda + \cosh \pi \lambda \cdot \sinh \pi \lambda) b_1 = \frac{1}{\lambda} \frac{\rho}{(1+4\lambda^2)^2} \cosh \pi \lambda - \frac{f\pi^2}{32} \left\{ \frac{1}{\lambda^4} - \frac{1}{(1+\lambda^2)^2} \right\} \sinh \pi \lambda$$

$$b_1 = \frac{\frac{\rho}{\lambda(1+4\lambda^2)^2} \frac{\cosh \pi \lambda}{\pi \lambda} - \frac{f\pi^2}{32} \left\{ \frac{1}{\lambda^4} - \frac{1}{(1+\lambda^2)^2} \right\} \frac{\sinh \pi \lambda}{\pi \lambda}}{1 + \frac{\sinh 2\pi \lambda}{2\pi \lambda}}$$

$$a_1 = \frac{\rho}{\lambda(1+4\lambda^2)^2} \frac{1}{\sinh \pi \lambda} - \frac{\sinh \pi \lambda + \pi \lambda \cosh \pi \lambda}{\sinh \pi \lambda} b_1$$

$$= \frac{\rho}{\lambda(1+4\lambda^2)^2} \frac{1}{\sinh \pi \lambda} - \frac{(\sinh \pi \lambda + \pi \lambda \cosh \pi \lambda) \left[\frac{\rho}{\lambda(1+4\lambda^2)^2} \cosh \pi \lambda - \frac{f\pi^2}{32} \left\{ \frac{1}{\lambda^4} - \frac{1}{(1+\lambda^2)^2} \right\} \right]}{\sinh \pi \lambda (\pi \lambda + \sinh \pi \lambda \cosh \pi \lambda)}$$

$$= \frac{-\frac{\rho \pi \lambda}{\lambda(1+4\lambda^2)^2} \sinh^2 \pi \lambda + (\sinh \pi \lambda + \pi \lambda \cosh \pi \lambda) \sinh \pi \lambda \frac{f\pi^2}{32} \left\{ \frac{1}{\lambda^4} - \frac{1}{(1+\lambda^2)^2} \right\}}{\sinh \pi \lambda (\pi \lambda + \sinh \pi \lambda \cosh \pi \lambda)}$$

$$a_1 = \frac{-\frac{\rho}{\lambda(1+4\lambda^2)^2} \sinh \pi \lambda + \left(\frac{\sinh \pi \lambda}{\pi \lambda} + \cosh \pi \lambda \right) \frac{f\pi^2}{32} \left\{ \frac{1}{\lambda^4} - \frac{1}{(1+\lambda^2)^2} \right\}}{1 + \frac{\sinh 2\pi \lambda}{2\pi \lambda}}$$

$$\cosh 2\pi \cdot a_2 + 2\pi\lambda \sinh 2\pi\lambda \cdot b_2 = \frac{1}{\lambda^4} \frac{f\pi^2}{512}$$

$$\sinh 2\pi\lambda \cdot a_2 + (\cosh 2\pi\lambda + 2\pi\lambda \cosh 2\pi\lambda) \cdot b_2 = 0$$

$$b_2 = - \frac{\frac{1}{\lambda^4} \frac{f\pi^2}{512} \cdot \frac{\sinh 2\pi\lambda}{2\pi\lambda}}{1 + \frac{\sinh 4\pi\lambda}{4\pi\lambda}}$$

$$a_2 = - \frac{\frac{\sinh 2\pi\lambda}{2\pi\lambda} + \cosh 2\pi\lambda}{\frac{\sinh 2\pi\lambda}{2\pi\lambda}} b_2$$

$$a_2 = + \frac{\frac{1}{\lambda^4} \frac{f\pi^2}{512} \cdot \left(\cosh 2\pi\lambda + \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right)}{1 + \frac{\sinh 4\pi\lambda}{4\pi\lambda}}$$

Terms depends upon a_1, b_1 , linearly

$$\begin{aligned}
 & - \frac{\pi^2}{64} \lambda^4 \left[(a_1 - b_1) \frac{\sinh \pi \lambda}{\pi \lambda} + b_1 \cosh \pi \lambda \right] \\
 & + \frac{1}{\pi} \frac{\cosh \pi \lambda}{(1+4\lambda^2)^3} \left\{ 16 \lambda^6 a_1 - 4 \lambda^6 (a_1 + 2b_1) - 32 \lambda^8 (a_1 + b_1) \right\} \\
 & + \left\{ \frac{2 \lambda \sinh \pi \lambda}{(1+4\lambda^2)^3} - \frac{16 \lambda^2 \cosh \pi \lambda}{\pi (1+4\lambda^2)^4} \right\} \left\{ 8 \lambda^6 b_1 - 2 \lambda^6 b_1 \right\} \\
 & - \left\{ \frac{2 \lambda \sinh \pi \lambda}{(1+4\lambda^2)^3} + \frac{2(1-4\lambda^2) \cosh \pi \lambda}{\pi (1+4\lambda^2)^4} \right\} \left\{ 16 \lambda^8 b_1 \right. \\
 & \left. + \frac{\pi^2}{64} \frac{1}{\pi} \frac{\sinh \pi \lambda}{(1+\lambda^2)^3} \left\{ \lambda^6 a_1 - \lambda^6 (a_1 + 2b_1) + 2 \lambda^6 (a_1 + b_1) \right\} \right\} \\
 & + \frac{\pi^2}{64} \left[\left\{ - \frac{\lambda^2 \cosh \pi \lambda}{(1+\lambda^2)^3} + \frac{\lambda(1-\lambda^2) \sinh \pi \lambda}{\pi (1+\lambda^2)^4} \right\} \left\{ \lambda^6 b_1 - \lambda^6 b_1 \right\} \right. \\
 & \left. + \left\{ \frac{\lambda^2 \cosh \pi \lambda}{(1+\lambda^2)^3} - \frac{2 \lambda^3 \sinh \pi \lambda}{\pi (1+\lambda^2)^4} \right\} \left\{ 2 \lambda^6 b_1 \right\} \right]
 \end{aligned}$$

$$\begin{aligned}
 & - \frac{\pi^2}{64} \lambda^4 \left[(a_1 - b_1) \frac{\sinh \pi \lambda}{\pi \lambda} + b_1 \cosh \pi \lambda \right] \\
 & - \frac{4 \lambda^6 \cosh \pi \lambda}{\pi (1+4\lambda^2)^3} (a_1 + 2b_1) \\
 & - \frac{4 \lambda^8 \sinh \pi \lambda}{(1+4\lambda^2)^2} b_1 \\
 & + \frac{\pi^2}{64} \frac{\lambda^2 \sinh \pi \lambda}{\pi (1+\lambda^2)^2} a_1 \\
 & + \frac{\pi^2}{64} \left[\frac{\lambda^8 \cosh \pi \lambda}{(1+\lambda^2)^2} b_1 - \frac{\lambda^2 \sinh \pi \lambda}{\pi (1+\lambda^2)^2} b_1 \right]
 \end{aligned}$$

Terms depends linearly upon a_1, b_1

$$- \frac{\pi^2}{64} \lambda^4 \left[(a_2 - b_2) \frac{\sinh \pi \lambda}{2 \pi \lambda} + b_2 \cosh \pi \lambda \right]$$

Terms in second order of a_1, b_1

$$\begin{aligned}
 & \frac{\lambda^8}{4} \left[\left\{ \frac{1}{2} + \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right\} (a_1^2 + a_1^2 + 4a_1b_1 + 4b_1^2) + \frac{1}{2} \left\{ \cosh 2\pi\lambda - \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right\} (a_1b_1 + a_1b_1 + 2b_1^2) \right. \\
 & \quad \left. + \left\{ -\frac{\pi^2\lambda^2}{6} + \frac{1}{8} \left[(4\pi^2\lambda^2 + 2) \frac{\sinh 2\pi\lambda}{2\pi\lambda} - 2 \cosh 2\pi\lambda \right] \right\} (b_1^2 + b_1^2) \right. \\
 & \quad \left. + \left\{ -\frac{1}{2} + \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right\} (2a_1^2 + 4a_1b_1 + 2b_1^2) + \frac{1}{2} \left\{ \cosh 2\pi\lambda - \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right\} (2a_1b_1 + 2b_1^2) \right. \\
 & \quad \left. + \left\{ \frac{\pi^2\lambda^2}{6} + \frac{1}{8} \left[(4\pi^2\lambda^2 + 2) \frac{\sinh 2\pi\lambda}{2\pi\lambda} - 2 \cosh 2\pi\lambda \right] \right\} (2b_1^2) \right] \\
 & = \frac{\lambda^8}{4} \left[4 \left(\frac{\sinh 2\pi\lambda}{2\pi\lambda} \right) (a_1 + b_1)^2 + 2b_1^2 \left(\frac{1}{2} + \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right) + 2 \left\{ \cosh 2\pi\lambda - \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right\} (a_1b_1 + b_1^2) \right. \\
 & \quad \left. + \frac{1}{2} b_1^2 \left\{ (4\pi^2\lambda^2 + 2) \frac{\sinh 2\pi\lambda}{2\pi\lambda} - 2 \cosh 2\pi\lambda \right\} \right] \\
 & = \frac{\lambda^8}{4} \left[4 \left(\frac{\sinh 2\pi\lambda}{2\pi\lambda} \right) a_1^2 + \left\{ 3 \left(\frac{\sinh 2\pi\lambda}{2\pi\lambda} \right) + \cosh 2\pi\lambda \right\} 2a_1b_1 + \left\{ 1 + (2\pi^2\lambda^2 + 5) \frac{\sinh 2\pi\lambda}{2\pi\lambda} + \cosh 2\pi\lambda \right\} b_1^2 \right]
 \end{aligned}$$

Terms in second order of a_2, b_2

$$4\lambda^6 \left[4 \left(\frac{\sinh 4\pi\lambda}{4\pi\lambda} \right) a_2^2 + \left\{ 3 \left(\frac{\sinh 4\pi\lambda}{4\pi\lambda} \right) + \cosh 4\pi\lambda \right\} 2a_2b_2 + \left\{ 1 + (8\pi^2\lambda^2 + 5) \frac{\sinh 4\pi\lambda}{4\pi\lambda} + \cosh 4\pi\lambda \right\} b_2^2 \right]$$

Terms independent of a_1, b_1, a_2, b_2

$$\left\{ \frac{1^2}{4(12)^2} + \frac{\lambda^4}{4(32)^2} + \frac{1}{8(32)^2} \frac{\lambda^4}{(1+\lambda^2)^2} (-f\pi^2)^2 + \frac{2\lambda^4}{(1+4\lambda^2)^2} \right\}$$

$$-\frac{f\pi^2}{64}\lambda^4 \left[(a_2 - b_2) \frac{\sinh 2\pi\lambda}{2\pi\lambda} + b_2 \cosh 2\pi\lambda \right] = -\frac{(f\pi^2)^2}{64 \times 512} \left[\frac{\left(\cosh 2\pi\lambda + \frac{2 \sinh 2\pi\lambda}{2\pi\lambda} \right) \frac{\sinh 2\pi\lambda}{2\pi\lambda} - \frac{\sinh 2\pi\lambda}{2\pi\lambda} \cosh 2\pi\lambda}{1 + \frac{\sinh 4\pi\lambda}{4\pi\lambda}} \right]$$

$$= -\frac{(f\pi^2)^2}{32 \times 512} \frac{\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda} \right)^2}{1 + \frac{\sinh 4\pi\lambda}{4\pi\lambda}}$$

$$4\lambda^6 \left[4 \left(\frac{\sinh 4\pi\lambda}{4\pi\lambda} \right) a_2^2 + \left\{ 3 \left(\frac{\sinh 4\pi\lambda}{4\pi\lambda} \right) + \cosh 4\pi\lambda \right\} 2a_2b_2 + \left\{ 1 + (8\pi^2\lambda^2 + 5) \frac{\sinh 4\pi\lambda}{4\pi\lambda} + \cosh 4\pi\lambda \right\} b_2^2 \right]$$

$$= \frac{\left(\frac{\pi^2}{256} \right)^2 \frac{\sinh 2\pi\lambda}{2\pi\lambda}}{\left(1 + \frac{\sinh 4\pi\lambda}{4\pi\lambda} \right)^2} \left[4 \cosh 2\pi\lambda \left\{ \cosh 2\pi\lambda + \frac{\sinh 6\pi\lambda}{2\pi\lambda} \right\}^2 - 2 \left\{ 3 \left(\frac{\sinh 4\pi\lambda}{4\pi\lambda} \right) + \cosh 4\pi\lambda \right\} \left\{ \cosh 2\pi\lambda + \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right\} \right. \\ \left. + \frac{\sinh 2\pi\lambda}{2\pi\lambda} \left\{ 1 + (8\pi^2\lambda^2 + 5) \frac{\sinh 4\pi\lambda}{4\pi\lambda} + \cosh 4\pi\lambda \right\} \right]$$

$$= \frac{\left(\frac{\pi^2}{256} \right)^2 \frac{\sinh 2\pi\lambda}{2\pi\lambda}}{\left(1 + \frac{\sinh 4\pi\lambda}{4\pi\lambda} \right)^2} \left[4 \cosh 2\pi\lambda \left\{ \frac{1}{2} \left(\cosh 4\pi\lambda + 1 \right) + 2 \frac{\sinh 4\pi\lambda}{4\pi\lambda} + \left(\frac{\sinh 2\pi\lambda}{2\pi\lambda} \right)^2 \right\} \right. \\ \left. + 6 \frac{\sinh 4\pi\lambda}{4\pi\lambda} \cdot \cosh 2\pi\lambda - 2 \cosh 2\pi\lambda \cosh 2\pi\lambda - 6 \frac{\sinh 4\pi\lambda}{4\pi\lambda} \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right. \\ \left. - 2 \cosh 4\pi\lambda \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right]$$

$$+ \frac{\sinh 2\pi\lambda}{2\pi\lambda} + (8\pi^2\lambda^2 + 5) \frac{\sinh 2\pi\lambda}{2\pi\lambda} \frac{\sinh 4\pi\lambda}{4\pi\lambda} + \frac{\sinh 2\pi\lambda}{2\pi\lambda} \cosh 4\pi\lambda \Big]$$

$$= \frac{\left(\frac{\pi^2}{256} \right)^2 \left(\frac{\sinh 2\pi\lambda}{2\pi\lambda} \right)}{\left(1 + \frac{\sinh 4\pi\lambda}{4\pi\lambda} \right)^2} \left\{ (8\pi^2\lambda^2 + 3) \frac{\sinh 4\pi\lambda}{4\pi\lambda} \frac{\sinh 2\pi\lambda}{2\pi\lambda} + 2 \left(-\cosh 2\pi\lambda + \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right) \right\}$$

The terms quadratic in a_1, b_1 , can be divided into three parts:

$$\frac{\left(\frac{16}{64}\right)^2 \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\}^2 \left(\frac{\sinh \pi \lambda}{\pi \lambda} \right)^2}{\left(1 + \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right)^2} \left\{ (2\lambda^2 \pi^2 + 3) \frac{\sinh \pi \lambda}{\pi \lambda} \frac{\sinh 2\pi \lambda}{2\pi \lambda} + 2 \left(\cosh \pi \lambda + \frac{\sinh \pi \lambda}{\pi \lambda} \right) \right\}$$

$$\frac{\left(\frac{16}{16}\right)^2 \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\}^2 \frac{\lambda^4}{\lambda(1+4\lambda^2)^2 \pi \lambda} \left[\frac{1}{4} \left(\frac{\sinh 2\pi \lambda}{\pi \lambda} \right)^2 \right] \left\{ -2 \sinh \pi \lambda \left(\frac{\sinh \pi \lambda}{\pi \lambda} + \cosh \pi \lambda \right) \right\}}{\left(1 + \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right)^2}$$

$$+ 2 \left\{ 3 \left(\frac{\sinh 2\pi \lambda}{2\pi \lambda} + \cosh 2\pi \lambda \right) \left[\sinh \pi \lambda \frac{\sinh \pi \lambda}{2\pi \lambda} + \frac{\sinh 2\pi \lambda}{2\pi \lambda} + \cosh \pi \lambda \cosh \pi \lambda \right] \right\}$$

$$- 2 \left\{ 1 + (2\pi^2 \lambda^2 + 5) \frac{\sinh 2\pi \lambda}{2\pi \lambda} + \cosh 2\pi \lambda \right\} \left\{ \frac{\sinh \pi \lambda}{\pi \lambda} \frac{\cosh \pi \lambda}{\pi \lambda} \right\}$$

$$= \frac{\left(\frac{16}{16}\right)^2 \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\}^2 \frac{\lambda^2}{\pi(1+4\lambda^2)^2}}{\left(1 + \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right)^2} \left[-4 \left(\frac{\sinh 2\pi \lambda}{2\pi \lambda} \right) \left(\cosh 2\pi \lambda - 1 \right) - 4 \sinh 2\pi \lambda \sinh \pi \lambda \cosh \pi \lambda \right. \\ \left. + 3 \left(\frac{\sinh 2\pi \lambda}{2\pi \lambda} \right) \left(\cosh 2\pi \lambda - 1 \right) + \cosh 2\pi \lambda \left(\cosh 2\pi \lambda - 1 \right) + 6 \left(\frac{\sinh 2\pi \lambda}{2\pi \lambda} \right)^2 + 3 \left(\frac{\sinh 2\pi \lambda}{2\pi \lambda} \right) \left(\cosh 2\pi \lambda + 1 \right) \right. \\ \left. + 2 \cosh 2\pi \lambda \frac{\sinh 2\pi \lambda}{2\pi \lambda} + 2 \cosh 2\pi \lambda \cosh \pi \lambda \cosh \pi \lambda - 2 \frac{\sinh 2\pi \lambda}{2\pi \lambda} - 2 (2\pi^2 \lambda^2 + 5) \left(\frac{\sinh 2\pi \lambda}{2\pi \lambda} \right)^2 \right. \\ \left. - 2 \frac{\sinh 4\pi \lambda}{4\pi \lambda} \right]$$

$$= \frac{\left(\frac{1}{16}\right)^2 \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \frac{\lambda^2}{\pi(1+4\lambda^2)^2}}{\left(1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2} \left[-4 \left(\frac{\sinh 2\pi\lambda}{2\pi\lambda} \right) (\cosh 2\pi\lambda - 1) - (\sinh 2\pi\lambda)^2 + (\cosh 2\pi\lambda + 1) \right. \\ \left. + 6 \left(\frac{\sinh 2\pi\lambda}{2\pi\lambda} \right) \cosh 2\pi\lambda + (\cosh 2\pi\lambda)^2 - \cosh 2\pi\lambda \right. \\ \left. + 6 \left(\frac{\sinh 2\pi\lambda}{2\pi\lambda} \right)^2 + 2 \cosh 2\pi\lambda \left(\frac{\cosh 2\pi\lambda - 1}{2\pi\lambda} \right) + 2 \frac{\sinh 2\pi\lambda}{2\pi\lambda} - (2\pi^2\lambda^2 + 4) \left(\frac{\sinh 2\pi\lambda}{2\pi\lambda} \right)^2 - 2 \frac{\sinh 2\pi\lambda}{2\pi\lambda} \cosh 2\pi\lambda \right]$$

$$= \frac{\left(\frac{1}{16}\right)^2 \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \frac{\lambda^2}{\pi(1+4\lambda^2)^2}}{\left(1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2} \left[1 + \left\{ 4 + \cosh 2\pi\lambda - (2\pi^2\lambda^2 + 2) \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right\} \frac{\cosh 2\pi\lambda}{2\pi\lambda} \right]$$

$$\left[\frac{\left\{ \frac{4\lambda^4}{\lambda(1+4\lambda^2)^2} \right\}^2}{\left(1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2} \left[2 \left(\frac{\sinh 2\pi\lambda}{2\pi\lambda} \right) (\cosh 2\pi\lambda - 1) - 2 \left\{ 3 \left(\frac{\sinh 2\pi\lambda}{2\pi\lambda} \right) + \cosh 2\pi\lambda \right\} \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right. \right. \\ \left. \left. + \left\{ 1 + (2\pi^2\lambda^2 + 5) \frac{\sinh 2\pi\lambda}{2\pi\lambda} + \cosh 2\pi\lambda \right\} \frac{1}{2\pi\lambda^2} (\cosh 2\pi\lambda + 1) \right] \right]$$

$$= \frac{\left\{ \frac{4\lambda^4}{\lambda(1+4\lambda^2)^2} \right\}^2}{\left(1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2} \left[-2 \left(1 + 3 \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right) \frac{\sinh 2\pi\lambda}{2\pi\lambda} + \frac{\sinh 2\pi\lambda}{2\pi\lambda} (\cosh 2\pi\lambda + 1) \right. \\ \left. \left\{ 1 + 5 \frac{\sinh 2\pi\lambda}{2\pi\lambda} + \cosh 2\pi\lambda \right\} \frac{(\cosh 2\pi\lambda + 1)}{2\pi\lambda^2} \right]$$

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$$= \frac{\left\{ \frac{4\lambda^2}{(1+4\lambda^2)^2} \right\}^2}{\left(1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2} \left[\frac{\sinh 2\pi\lambda}{2\pi\lambda} \lambda^2 \left(\cosh 2\pi\lambda - 1 - 6 \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right) + \frac{1}{2\pi^2} \left(1 + 5 \frac{\sinh 2\pi\lambda}{2\pi\lambda} + \cosh 2\pi\lambda \right) \right] \cdot (\cosh 2\pi\lambda + 1)$$

Terms depends linearly on a_1, b_1 ,

$$\lambda^4 \frac{\pi^2}{64} \left[- \left\{ (a_1 - b_1) \frac{\sinh \pi\lambda}{\pi\lambda} + b_1 \cosh \pi\lambda \right\} + \frac{\lambda^3}{(1+\lambda^2)^2} \left\{ \frac{\sinh \lambda\pi}{\pi} a_1 + \lambda \cosh \pi\lambda b_1 - \frac{\sinh 2\pi}{\pi} b_1 \right\} \right] - \frac{4\lambda^6}{(1+4\lambda^2)^2} \left[\frac{\cosh \pi\lambda}{\pi} (a_1 + 2b_1) + \lambda \sinh \pi\lambda b_1 \right]$$

$$\frac{\left(\frac{\pi}{2}\right)^2}{32 \times 64} \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \left[-2 \left(\frac{\sinh \pi\lambda}{\pi\lambda} \right)^2 + \frac{\lambda^4}{(1+\lambda^2)^2} \right] \left\{ \frac{\sinh \lambda\pi}{\pi\lambda} \left(\frac{\sinh \pi\lambda}{\pi\lambda} + \cosh \pi\lambda \right) - \cosh \pi\lambda \cdot \frac{\sinh \pi\lambda}{\pi\lambda} \right. \\ \left. + \frac{\sinh \lambda\pi}{\pi\lambda} \cdot \frac{\sinh \pi\lambda}{\pi\lambda} \right\}$$

$$= \frac{\left(\frac{\pi}{2}\right)^2}{2 \times (32)^2} \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \left[-2 \left(\frac{\sinh \pi\lambda}{\pi\lambda} \right)^2 + \frac{\lambda^4}{(1+\lambda^2)^2} \right] \left[\frac{\sinh 2\pi\lambda}{2\pi\lambda} + \frac{\sinh \pi\lambda}{\pi\lambda} \right]$$

$$= - \frac{\left(\frac{\pi}{2}\right)^2}{(32)^2} \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \frac{\left(\frac{\sinh \pi\lambda}{\pi\lambda} \right)^2}{1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}}$$

$$\frac{1}{(1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda})} \left(\frac{1}{\pi^2} \right) \left[-\frac{\lambda^4}{8\lambda(1+\lambda^2)^2} \left\{ -\left(\cosh \pi\lambda - \frac{\cosh \pi\lambda}{\pi\lambda} \right) \frac{\sinh \pi\lambda}{\pi\lambda} + \left(\cosh \pi\lambda \right)^2 \right\} \right.$$

$$\left. + \frac{\lambda^3}{(1+\lambda^2)^2} \left[-\frac{(\sinh \lambda \pi)^2}{\pi} + \frac{1}{\pi} (\cosh \pi\lambda)^2 - \frac{1}{\pi} \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right] \right\}$$

$$- \frac{1}{8} \frac{\lambda^2}{(1+\lambda^2)^2} \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \left\{ \frac{\cosh \pi\lambda}{\pi} \left(\cosh \pi\lambda - \frac{\sinh \pi\lambda}{\pi\lambda} \right) - \frac{1}{\pi} (\sinh \pi\lambda)^2 \right\}$$

$$= \frac{1}{(1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda})} \left[\frac{\lambda^2}{8\pi(1+\lambda^2)^2} \left(-1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right) + \frac{\lambda^4}{(1+\lambda^2)^2} \left(1 - \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right) \right] \left\{ \right.$$

$$\left. - \frac{1}{8\pi} \frac{\lambda^2}{(1+\lambda^2)^2} \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \left\{ 1 - \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right\} \right\}$$

$$= - \frac{1}{(1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda})} \frac{1}{4\pi(1+\lambda^2)^2} \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \left\{ 1 - \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right\}$$

$$- \frac{\frac{32\lambda^6}{\lambda(1+4\lambda^2)^4}}{1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}} \left[\frac{\cosh \pi\lambda}{\pi} \left\{ -\sinh \pi\lambda + 2 \frac{\cosh \pi\lambda}{\pi\lambda} \right\} + \frac{\sinh \pi\lambda \cosh \pi\lambda}{\pi} \right]$$

$$= - \frac{32\lambda^4}{\pi^2(1+4\lambda^2)^2} \frac{(\cosh 2\pi\lambda + 1)}{1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}}$$

$$H_1(\lambda) = \frac{1^7}{4(12\lambda)^2} + \frac{\lambda^4}{4(32)^2} - \frac{\lambda^4}{8(32)^2(1+\lambda^2)^2} - \frac{1}{16(32)^2} \frac{(\sinh 2\pi\lambda)^2}{1 + \frac{\sinh 4\pi\lambda}{4\pi\lambda}}$$

$$+ \frac{1}{64(32)^2} \frac{\frac{\sinh 2\pi\lambda}{2\pi\lambda} \left(1 + \frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)^2}{\left(8\pi\lambda^2 \epsilon_3\right)^{-1} \frac{\sinh 2\pi\lambda}{2\pi\lambda} \frac{\sinh 4\pi\lambda}{4\pi\lambda}} \left\{ (\cosh 2\pi\lambda \ominus \frac{\sinh 2\pi\lambda}{2\pi\lambda}) \right\}$$

$$+ \frac{1}{4(32)^2} \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\}^2 \frac{\sinh \pi\lambda}{\pi\lambda} \frac{1}{\left(1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2} \left\{ (9\pi^2\lambda^2 \epsilon_3)^{-1} \frac{\sinh \pi\lambda}{\pi\lambda} \frac{\sinh 2\pi\lambda}{2\pi\lambda} + 2 \left(\cosh \pi\lambda \ominus \frac{\sinh \pi\lambda}{\pi\lambda} \right) \right\}$$

$$- \frac{1}{(32)^2} \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\}^2 \frac{1}{1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}} \frac{2}{\pi\lambda} \left(\frac{\sinh \pi\lambda}{\pi\lambda} \right)^2$$

$$H_2(\lambda) = \frac{1}{8\pi} \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \frac{\lambda^2}{(1+4\lambda^2)^2} \cdot \frac{[-1 + \{1 + \cosh 2\pi\lambda - (2\pi\lambda^2 + 2) \frac{\sinh 2\pi\lambda}{2\pi\lambda}\} \frac{\sinh 2\pi\lambda}{2\pi\lambda}]}{(1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda})^2}$$

$$- \frac{1}{4\pi} \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \frac{\lambda^2}{(1+4\lambda^2)^2} \cdot \frac{1 - \frac{\sinh 2\pi\lambda}{2\pi\lambda}}{1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}}$$

$$H_2(\lambda) = \frac{1}{8\pi} \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \frac{\lambda^2}{(1+4\lambda^2)^2} \cdot \frac{\frac{\sinh 2\pi\lambda}{2\pi\lambda} \left\{ 1 + \cosh 2\pi\lambda - 2\pi\lambda^2 \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right\} - 1}{\left(1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right)^2}$$

$$H_3^{(1)}(\lambda) = \frac{2\lambda^4}{(1+4\lambda^2)^2} + \left(\frac{4\lambda^2}{(1+4\lambda^2)^2} \right)^2 \left[\lambda^2 \frac{\sinh 2\pi\lambda}{2\pi\lambda} \left(\cosh 2\pi\lambda - 1 - 6 \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right) + \frac{1}{2\pi^2} \left(1 + 5 \frac{\sinh 2\pi\lambda}{2\pi\lambda} + \cosh 2\pi\lambda \right) \left(\cosh 2\pi\lambda + 1 \right) \right] \frac{1}{\left(1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right)^2}$$

$$- \frac{32}{\pi^2} \frac{\lambda^4}{(1+4\lambda^2)^4} \cdot \frac{(1 + \cosh 2\pi\lambda)}{1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}}$$

$$\kappa = 2 \left\{ 512 G_1 \theta_3 \left(\frac{\theta}{\epsilon} \right)^2 + G_3 \left(\frac{1}{\theta} + \frac{\lambda^2}{3} + \frac{2\lambda^4}{3} \right) \right\}^{\frac{1}{2}} + 24 G_2 \left(\frac{\theta}{\epsilon} \right) \quad \underline{\underline{596}}$$

$$\left(\frac{\theta}{\epsilon} \right)^2 = \left(\frac{\epsilon}{\theta} \right) \pi^2 \left\{ -\frac{8G_1}{G_3} \left(\frac{\theta}{\epsilon} \right)^2 + \frac{1}{G_3} \left(\frac{1}{512} + \frac{\lambda^2}{192} + \frac{\lambda^4}{96} \right) \right\}^{\frac{1}{2}} \frac{1}{\lambda^2} \quad \underline{\underline{!!!}} \dots$$

$$G_1(\lambda) = \frac{17}{65536} + \frac{\lambda^4}{4096} + \frac{1}{8192} \frac{\lambda^4}{(1+\lambda^2)^2} \\ + \frac{1}{65536} \frac{\frac{\sinh 2\pi\lambda}{2\pi\lambda}}{\left(1 + \frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)^2} \left\{ (8\pi^2\lambda^2 - 1) \frac{\sinh 2\pi\lambda}{2\pi\lambda} \frac{\sinh 4\pi\lambda}{4\pi\lambda} + 2 \left(\cosh 2\pi\lambda - \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right) \right\} \\ + \frac{1}{4096} \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \frac{\frac{\sinh \pi\lambda}{\pi\lambda}}{\left(1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2} \left\{ (2\pi^2\lambda^2 - 1) \frac{\sinh \pi\lambda}{\pi\lambda} \frac{\sinh 2\pi\lambda}{2\pi\lambda} + 2 \left(\cosh \pi\lambda - \frac{\sinh \pi\lambda}{\pi\lambda} \right) \right\}$$

$$G_2(\lambda) = \frac{1}{8\pi} \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \frac{1}{(1+4\lambda^2)^2} \frac{\frac{\sinh 2\pi\lambda}{2\pi\lambda} \left\{ 1 + \cosh 2\pi\lambda - 2\pi^2\lambda^2 \frac{\sinh 2\pi\lambda}{2\pi\lambda} \right\} - 1}{\left(1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}$$

$$G_3(\lambda) = \frac{2}{(1+4\lambda^2)^2} \\ + \left\{ \frac{4}{(1+4\lambda^2)^2} \right\} \frac{2 \left[\lambda^2 \frac{\sinh 2\pi\lambda}{2\pi\lambda} (\cosh 2\pi\lambda - 1 - 6 \frac{\sinh 2\pi\lambda}{2\pi\lambda}) + \frac{1}{2\pi^2} (-3 + \frac{\sinh 2\pi\lambda}{2\pi\lambda} + \cosh 2\pi\lambda (\cosh 2\pi\lambda + 1)) \right]}{\left(1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}$$

$$\frac{\sinh 2\pi}{2\pi} = 42.613218, \quad \cosh 2\pi = 267.24862$$

$$\frac{\sinh 4\pi}{4\pi} = 11409.473, \quad \cosh 4\pi = 143375.61$$

$$\frac{\sinh 8\pi}{8\pi} = 1635840500, \quad \cosh 8\pi = 41113157000$$

$$\text{for } \lambda = 1.000$$

$$\frac{400\pi\lambda}{\pi\lambda} = 3.676164$$

592

$$G_1(\lambda) = 0.000259399 + 0.000244141 + 0.00030518$$

$$+ 0.0000152588 \frac{42.613218}{11410.473^2} \left\{ 77.956835 \times 4 \quad 613218 \times 11409.473 + 2 \times 225.13540 \right\}$$

$$+ 0.000244141 \times 0.5625 \times \frac{3.676164}{43.613218^2} \left\{ 18.7392 \times 3.676164 \times 42.613218 + 2 \times 7.91584 \right\}$$

$$= 0.000259399 + 0.000244141 + 0.00030518$$

$$+ 0.0000152588 \frac{42.613218 \times 37902624}{11410.473^2} + 0.000244141 \times 0.5625 \times \frac{3.676164 \times 2951.360}{43.613218^2}$$

$$= 0.000259399 + 0.000244141 + 0.00030518 + 0.00009290 + 0.000463329$$

$$= \underline{\underline{0.00150168}}$$

$$G_2(\lambda) = 0.03978873 \times 0.75 \times 0.04 \frac{42.613218 \left\{ 268.74482 - 84.115121 \right\} - 1}{43.613218^2}$$

$$= -0.03978873 \times 0.75 \times 0.04 \frac{24399.916}{43.613218} = \underline{\underline{-0.0153076}}$$

$$G_3(\lambda) = 0.08$$

$$+ 0.0256 \frac{42.613218 \times 11.06930 + \frac{1}{2\pi^2} \times 307.36184 \times 268.74482}{43.613218^2}$$

$$= 0.08 + 0.0256 \frac{4656.4189}{43.613218^2}$$

$$= 0.08 + 0.062669 = \underline{\underline{0.142669}}$$

$$K = 2 \left\{ 0.110058 \left(\frac{f}{E} \right)^2 + 0.160503 \right\} - 0.367382 \left(\frac{f}{E} \right) \quad \lambda = 1.00 \quad \underline{52f}$$

$$K_0 = \underline{0.80126}$$

$$0.220116 \left(\frac{f}{E} \right) = 0.367382 \left\{ 0.110058 \left(\frac{f}{E} \right)^2 + 0.160503 \right\}$$

$$0.0484511 \left(\frac{f}{E} \right)^2 = 0.0216131$$

$$\frac{0.0148545}{0.0335966}$$

$$\left(\frac{f}{E} \right) = 0.644800$$

$$\left(\frac{f}{E} \right) = 0.80300$$

$$K_{\min} = \underline{0.66722}$$

$$\lambda = 0.5$$

$$\frac{\sinh 0.5\pi}{0.5\pi} = 1.46505,$$

$$\cosh 0.5\pi = 2.50920$$

$$\frac{\sinh \pi}{\pi} = 3.676164,$$

$$\cosh \pi = 11.5920$$

$$G_1(\lambda) = 0.000259399 + 0.000015259 + 0.000004863$$

$$+ 0.0000152588 \times \frac{3.676164}{43.613218^2} \left\{ 18.739209 \times 3.67614 \times 42.613218 + 2 \times 29.1554 \right\}$$

$$+ 0.000244141 \times 0.9216 \times \frac{1.46505}{4.676164^2} \left\{ 3.9348022 \times 1.46505 \times 3.676164 + 2 \times 1.04415 \right\}$$

$$= 0.000259399 + 0.000015259 + 0.000004863 + 0.0000152588 \times \frac{10849.7171}{43.613218^2}$$

$$+ 0.000244141 \times 0.9216 \times \frac{34.106680}{4.676164^2}$$

$$= 0.000259399 + 0.000015259 + 0.000004863 + 0.000007037 + 0.000350948$$

$$= \underline{0.000717526}$$

$$G_2(1) = 0.03978873 \times 0.96 \times 0.25 \times \frac{3.676164 \{12.5920 - 18.141142\} - 1}{4.676164^2} \quad \underline{\underline{529}}$$

$$= -0.03978873 \times 0.96 \times 0.25 \times \frac{21.39956}{4.676164^2} = \underline{\underline{-0.00934538}}$$

$$G_3(x) = 0.5 + \frac{-0.25 \times 3.676164 \times 11.46964 + 7.826085}{4.676164^2}$$

$$= 0.5 - \frac{2.715168}{4.676164^2} = \underline{\underline{0.375830}}$$

$$K = 2 \left\{ 0.138070 \left(\frac{f}{T}\right)^2 + 0.0939575 \right\}^{\frac{1}{2}} - 0.224289 \left(\frac{f}{T}\right) \quad \lambda = 0.5$$

$$K_0 = \underline{\underline{0.61305}}$$

$$0.276140 \left(\frac{f}{T}\right) = 0.224289 \left\{ 0.138070 \left(\frac{f}{T}\right)^2 + 0.0939575 \right\}^{\frac{1}{2}}$$

$$0.0762533 \left(\frac{f}{T}\right)^2 = 0.0047266$$

$$\frac{0.0069457}{0.0693026}$$

$$\left(\frac{f}{T}\right)^2 = 0.0651921$$

$$\left(\frac{f}{T}\right) = 0.261126$$

$$K_{min} = \underline{\underline{0.58446}}$$

$$\lambda = 1.5$$

$$0.222222$$

$$\log_{10}(e^{1.5\pi}) = 0.434294482 \times 1.5\pi = 2.0465646$$

$$e^{1.5\pi} = 111.31779,$$

$$e^{-1.5\pi} = 0.00898$$

$$\sinh 1.5\pi = 55.654405$$

$$\cosh 1.5\pi = 55.663385$$

$$\frac{\sinh 1.5\pi}{1.5\pi} = 11.810231$$

$$\log_{10}(e^{3.0\pi}) = 0.434294482 \times 9.4247781 = 4.0931291$$

$$e^{3\pi} = 19391.650, \quad e^{-3\pi} = 0.000$$

$$\sinh 3.0\pi \approx \cosh 3.0\pi = 6195.8250$$

$$\frac{\sinh 3\pi}{3\pi} = 657.39245$$

$$\sinh 6\pi \approx \cosh 6\pi = 76776495$$

$$\frac{\sinh 6\pi}{6\pi} = 4073119.5$$

$$\frac{\sinh 1.5\pi}{1.5\pi} = 11.810231$$

$$\cosh 1.5\pi = 55.663385$$

$$\frac{\sinh 3\pi}{3\pi} = 657.39245$$

$$\cosh 3\pi = 6195.8250$$

$$\frac{\sinh 6\pi}{6\pi} = 4073119.5$$

$$\cosh 6\pi = 76776495$$

$$G_1(\lambda) = 0.000259399 + 0.001235962 + 0.000058507$$

$$+ 0.0000152588 \times \frac{657.39245}{4073120.5^2} \left\{ 176.65288 \times 657.39245 \times 4073119.5 + 11076.455 \right\}$$

$$+ 0.000244141 \times 0.27113897 \times \frac{11.810231}{657.39245^2} \left\{ 43.413220 \times 11.810231 \times 657.39245 + 87.706306 \right\}$$

$$= 0.000259399 + 0.001235962 + 0.000058507$$

$$+ 0.0000152588 \times \frac{31095.956}{40731205^2} + 0.000244141 \times 0.27113897 \times \frac{398.18032}{65739245^2}$$

$$= 0.000259399 + 0.001235962 + 0.000058507 + 0.000285954 + 0.000608066 = \underline{\underline{0.002447861}}$$

$$G_2(\lambda) = -0.03978873 \times 0.52071006 \times 0.01 \frac{657.39745 \times 23000.313}{657.39745^2} + 1 \frac{53}{53}$$

$$= -0.03978873 \times 0.52071006 \times 0.01 \times 34.880726 = \underline{\underline{-0.00722673}}$$

$$G_3(\lambda) = 0.02 + 0.0016 \frac{2.25 \times 657.39745 \times 2250.4400 + 2181916.9}{657.39745^2}$$

$$= 0.02 + 0.0016 \frac{551.06423}{43.348720}$$

$$= 0.02 + 0.0203398 = \underline{\underline{0.0403398}}$$

$$\frac{f}{g} + \frac{\lambda^2}{3} + \frac{2\lambda^4}{3} = \frac{f}{g} + \frac{1}{3}\lambda^2(1+2\lambda^2) = 0.125 + 4.125 = \underline{\underline{4.2500}}$$

$$K = 2 \left\{ 0.0505582 \left(\frac{f}{E}\right)^2 + 0.171444 \right\}^{\frac{1}{2}} - 0.173442 \left(\frac{f}{E}\right)$$

$$K_0 = \underline{\underline{0.82812}} \quad 0.1011164 \left(\frac{f}{E}\right) = 0.173442 \left\{ 0.0505582 \left(\frac{f}{E}\right)^2 + 0.171444 \right\}$$

$$\frac{0.01022453}{0.00152090} \left(\frac{f}{E}\right)^2 = 0.005157408$$

$$\left(\frac{f}{E}\right)^2 = 0.592558$$

$$\left(\frac{f}{E}\right) = 0.76978$$

$$K_{min} = \underline{\underline{0.76405}}$$

$$\frac{u}{R} = \frac{1}{2} \left(\frac{a}{R} \right)^2 \left[1 - \left(\frac{x}{a} \right)^2 - \frac{f}{4} \left(1 + \cos \frac{\pi x}{a} \right) \left(1 + \cos \frac{\pi y}{b} \right) \right]$$

$$\frac{u_0}{R} = \frac{1}{2} \left(\frac{a}{R} \right)^2 \left[1 - \left(\frac{x}{a} \right)^2 \right]$$

$$\frac{\partial u}{\partial x} = \left(\frac{a}{R} \right) \left\{ - \left(\frac{x}{a} \right) + \frac{f}{8} \pi \sin \frac{\pi x}{a} \left(1 + \cos \frac{\pi y}{b} \right) \right\}$$

$$\frac{\partial u}{\partial y} = \left(\frac{a}{R} \right) \left\{ + \frac{f}{8} \pi \lambda \left(1 + \cos \frac{\pi x}{a} \right) \sin \frac{\pi y}{b} \right\}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{R} \left\{ -1 + \frac{f}{8} \pi^2 \cos \frac{\pi x}{a} \left(1 + \cos \frac{\pi y}{b} \right) \right\}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{R} \left\{ + \frac{f}{8} \pi^2 \lambda^2 \left(1 + \cos \frac{\pi x}{a} \right) \cos \frac{\pi y}{b} \right\}$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{1}{R} \left\{ - \frac{f}{8} \pi^2 \lambda \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \right\}$$

$$\begin{aligned} \Delta F = \frac{1}{R^2} E \frac{f \pi^2 \lambda^2}{8} & \left\{ \frac{f \pi^2}{32} \left[\left(1 - \cos \frac{2\pi x}{a} \right) \left(1 - \cos \frac{2\pi y}{b} \right) - \left(1 + 2 \cos \frac{\pi x}{a} + \cos \frac{2\pi x}{a} \right) \right. \right. \\ & \left. \left. \left(1 + 2 \cos \frac{\pi y}{b} + \cos \frac{2\pi y}{b} \right) \right] \right. \\ & \left. + \cos \frac{\pi y}{b} + \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} \right\} \end{aligned}$$

$$\begin{aligned} = \frac{E}{R^2} \frac{f \pi^2 \lambda^2}{8} & \left\{ \frac{f \pi^2}{32} \left[-2 \cos \frac{2\pi x}{a} - 2 \cos \frac{2\pi y}{b} - 2 \cos \frac{\pi x}{a} - 2 \cos \frac{\pi y}{b} - 4 \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} \right. \right. \\ & \left. \left. - 2 \cos \frac{2\pi x}{a} \cos \frac{\pi y}{b} - 2 \cos \frac{\pi x}{a} \cos \frac{2\pi y}{b} \right] \right. \\ & \left. + \cos \frac{\pi y}{b} + \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} \right\} \end{aligned}$$

$$\begin{aligned} = \frac{E}{R^2} \frac{f \pi^2 \lambda^2}{8} & \left[\left(1 - \frac{f \pi^2}{16} \right) \cos \frac{\pi y}{b} + \left(1 - \frac{f \pi^2}{8} \right) \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} \right. \\ & \left. - \frac{f \pi^2}{16} \left\{ \cos \frac{2\pi x}{a} + \cos \frac{2\pi y}{b} + \cos \frac{\pi x}{a} + \cos \frac{\pi y}{b} \cos \frac{2\pi y}{b} + \cos \frac{2\pi x}{a} \cos \frac{\pi y}{b} \right\} \right] \end{aligned}$$

The particular integral is

$$\begin{aligned}
 F^{(1)} &= \frac{E - \frac{1}{2} \pi^2 \lambda^2}{R^2} \left[\left(1 - \frac{f \pi^2}{16}\right) \frac{\cos \frac{\pi x}{b}}{\left(\frac{\pi}{b}\right)^4} + \left(1 - \frac{f \pi^2}{8}\right) \frac{\cos \frac{\pi x}{a} \cos \frac{\pi y}{b}}{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2} \right] \\
 &- \frac{f \pi^2}{16} \left\{ \frac{\cos \frac{2 \pi x}{a}}{\left(\frac{2 \pi}{a}\right)^4} + \frac{\cos \frac{2 \pi y}{b}}{\left(\frac{2 \pi}{b}\right)^4} + \frac{\cos \frac{\pi x}{a}}{\left(\frac{\pi}{a}\right)^4} + \frac{\cos \frac{\pi y}{b}}{\left(\frac{\pi}{b}\right)^4} + \frac{\cos \frac{2 \pi x}{a} \cos \frac{2 \pi y}{b}}{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2} + \frac{\cos \frac{2 \pi x}{a} \cos \frac{\pi y}{b}}{\left(\frac{2 \pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2} \right\} \\
 F &= E \left(\frac{a^2}{R} \right) \frac{f \lambda^2}{8} \frac{1}{(a)^2} \left[\left(1 - \frac{f \pi^2}{16}\right) \frac{1}{\lambda^4} \cos \frac{\pi x}{b} + \left(1 - \frac{f \pi^2}{8}\right) \frac{\cos \frac{\pi x}{a} \cos \frac{\pi y}{b}}{(1 + \lambda^2)^2} \right. \\
 &- \frac{f \pi^2}{16} \left\{ \cos \frac{\pi x}{a} + \frac{1}{16} \cos \frac{2 \pi x}{a} + \frac{1}{16 \lambda^4} \cos \frac{2 \pi y}{b} + \frac{\cos \frac{\pi x}{a} \cos \frac{2 \pi y}{b}}{(1 + 4 \lambda^2)^2} + \frac{\cos \frac{2 \pi x}{a} \cos \frac{\pi y}{b}}{(4 + \lambda^2)^2} \right\} \\
 &+ a_0 \left(\frac{\pi x}{a} \right)^2 + \left\{ a_1 \cosh \frac{\pi \lambda x}{a} + b_1 \left(\frac{\pi \lambda x}{a} \right) \sinh \frac{\pi \lambda x}{a} \right\} \cos \frac{\pi y}{b} \\
 &+ \left\{ a_2 \cosh \frac{2 \pi \lambda x}{a} + b_2 \left(\frac{2 \pi \lambda x}{a} \right) \sinh \frac{2 \pi \lambda x}{a} \right\} \cos \frac{2 \pi y}{b} \left. \right]
 \end{aligned}$$

$$\begin{aligned}
 \phi = \frac{\partial V}{\partial \psi} &= E \left(\frac{a}{R} \right)^2 \frac{1}{\rho} \left[\left(\frac{1}{16} \lambda^2 - 1 \right) \frac{1}{\lambda^2} \cos \frac{\pi x}{b} + \left(\frac{1}{8} \lambda^2 - 1 \right) \frac{\lambda^2}{(1+\lambda^2)^2} \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} \right. \\
 &+ \frac{1}{16} \lambda^2 \left\{ \frac{1}{4\lambda^2} \cos \frac{2\pi x}{b} + \frac{4\lambda^2}{(1+4\lambda^2)^2} \cos \frac{\pi x}{a} \cos \frac{2\pi y}{b} + \frac{\lambda^2}{(4+\lambda^2)^2} \cos \frac{2\pi x}{a} \cos \frac{\pi y}{b} \right\} \\
 &- \lambda^2 \left\{ a_1 \cosh \frac{\pi x}{a} + b_1 \left(\frac{\pi x}{a} \right) \sinh \frac{\pi x}{a} \right\} \cos \frac{\pi y}{b} \\
 &- 4\lambda^2 \left\{ a_2 \cosh \frac{2\pi x}{a} + b_2 \left(\frac{2\pi x}{a} \right) \sinh \frac{2\pi x}{a} \right\} \cos \frac{2\pi y}{b} \left. \right] \\
 &= E \left(\frac{a}{R} \right)^2 \frac{1}{\rho} \left\{ \left[\left(\frac{1}{16} \lambda^2 - 1 \right) \frac{1}{\lambda^2} + \left(\frac{1}{8} \lambda^2 - 1 \right) \frac{\lambda^2}{(1+\lambda^2)^2} \cos \frac{\pi x}{a} + \frac{1}{16} \frac{\lambda^2}{(4+\lambda^2)^2} \cos \frac{2\pi x}{a} \right. \right. \\
 &\quad \left. \left. - \lambda^2 \left\{ a_1 \cosh \frac{\pi x}{a} + b_1 \left(\frac{\pi x}{a} \right) \sinh \frac{\pi x}{a} \right\} \right] \cos \frac{\pi y}{b} \right. \\
 &\quad \left. + \left[\frac{1}{16} \lambda^2 \left\{ \frac{1}{4\lambda^2} + \frac{4\lambda^2}{(1+4\lambda^2)^2} \cos \frac{\pi x}{a} \right\} - 4\lambda^2 \left\{ a_2 \cosh \frac{2\pi x}{a} + b_2 \left(\frac{2\pi x}{a} \right) \sinh \frac{2\pi x}{a} \right\} \right] \cos \frac{2\pi y}{b} \right\}
 \end{aligned}$$

$$\begin{aligned}
 a_1 \cosh \pi \lambda + b_1 (\pi \lambda) \sinh \pi \lambda &= \frac{1}{\lambda^4} \left[\left(\frac{1}{16} \lambda^2 - 1 \right) - \left(\frac{1}{8} \lambda^2 - 1 \right) \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{1}{16} \frac{\lambda^4}{(4+\lambda^2)^2} \right] \\
 a_2 \cosh 2\pi \lambda + b_2 (2\pi \lambda) \sinh 2\pi \lambda &= \frac{1}{16\lambda^4} \left[\frac{1}{16} \lambda^2 \left\{ 1 - \frac{16\lambda^4}{(1+4\lambda^2)^2} \right\} \right]
 \end{aligned}$$

Conditio for free edge!

$$T_{xy} = -\frac{\partial^2 F}{\partial x \partial y} = E \left(\frac{a^2}{R} \right) \frac{\pm \lambda^2}{8} \left[\left(\frac{\lambda^2}{8} - 1 \right) \frac{\lambda}{(1+\lambda^2)^2} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + \frac{\lambda^2}{16} \left(\frac{\lambda^2}{(1+4\lambda^2)^2} \sin \frac{\pi x}{a} + \frac{\lambda^2}{(1+4\lambda^2)^2} \sin \frac{\pi y}{b} \right) \right. \\ \left. + \frac{2\lambda}{(4+\lambda^2)^2} \sin \frac{2\pi x}{a} \sin \frac{\pi y}{b} \right]$$

$$+ \lambda^2 \left\{ (a_1 + b_1) \sinh \frac{\pi \lambda x}{a} + b_1 \left(\frac{\pi \lambda x}{a} \right) \cosh \frac{\pi \lambda x}{a} \right\} \sin \frac{\pi y}{b}$$

$$+ 4\lambda^2 \left\{ (a_2 + b_2) \sinh \frac{2\pi \lambda x}{a} + b_2 \left(\frac{2\pi \lambda x}{a} \right) \cosh \frac{2\pi \lambda x}{a} \right\} \sin \frac{2\pi y}{b} \left. \right]$$

$$= E \left(\frac{a^2}{R} \right) \frac{\pm \lambda^2}{8} \left[\left(\frac{\lambda^2}{8} - 1 \right) \frac{\lambda}{(1+\lambda^2)^2} \sin \frac{\pi x}{a} + \frac{\lambda^2}{16} \frac{2\lambda}{(4+\lambda^2)^2} \sin \frac{2\pi x}{a} \right. \\ \left. + \lambda^2 \left\{ (a_1 + b_1) \sinh \frac{\pi \lambda x}{a} + b_1 \left(\frac{\pi \lambda x}{a} \right) \cosh \frac{\pi \lambda x}{a} \right\} \sin \frac{\pi y}{b} \right. \\ \left. + \left[\frac{\lambda^2}{16} \frac{2\lambda}{(1+4\lambda^2)^2} \sin \frac{\pi x}{a} + 4\lambda^2 \left\{ (a_2 + b_2) \sinh \frac{2\pi \lambda x}{a} + b_2 \left(\frac{2\pi \lambda x}{a} \right) \cosh \frac{2\pi \lambda x}{a} \right\} \sin \frac{2\pi y}{b} \right] \right]$$

$$(a_1 + b_1) \sinh \pi \lambda + b_1 (\pi \lambda) \cosh \pi \lambda = 0$$

$$(a_2 + b_2) \sinh 2\pi \lambda + b_2 (2\pi \lambda) \cosh 2\pi \lambda = 0$$

$$\begin{aligned} \tilde{G} = \frac{\partial^2 F}{\partial \lambda^2} &= E \left(\frac{a}{R} \right)^2 \frac{f \lambda^2}{g} \left[\left(\frac{f \lambda^2}{g} - 1 \right) \frac{1}{(1+\lambda^2)^2} \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} + \frac{f \lambda^2}{16} \left\{ \cos \frac{\pi x}{a} + \frac{1}{4} \cos \frac{2\pi x}{a} + \frac{\cos \frac{\pi x}{a} \cos \frac{2\pi y}{b}}{(1+4\lambda^2)^2} + \right. \right. \\ &+ \left. \frac{4}{(4\lambda^2)^2} \cos \frac{2\pi x}{a} \cos \frac{\pi y}{b} \right\} + 2a_0 + \lambda^2 \left\{ (a_1 + 2b_1) \cosh \frac{\pi x}{a} + b_1 \left(\frac{\pi x}{a} \right) \sinh \frac{\pi x}{a} \right\} \cos \frac{\pi y}{b} + \\ &+ 4\lambda^2 \left\{ (a_2 + 2b_2) \cosh \frac{2\pi x}{a} + b_2 \left(\frac{2\pi x}{a} \right) \sinh \frac{2\pi x}{a} \right\} \cos \frac{2\pi y}{b} \left. \right] \end{aligned}$$

$$\begin{aligned} &= E \left(\frac{a}{R} \right)^2 \frac{f \lambda^2}{g} \left[\frac{f \lambda^2}{16} \cos \frac{\pi x}{a} + \frac{f \lambda^2}{64} \cos \frac{2\pi x}{a} + 2a_0 \right. \\ &+ \left. \left\{ \left(\frac{f \lambda^2}{g} - 1 \right) \frac{\cos \frac{\pi x}{a}}{(1+\lambda^2)^2} + \frac{f \lambda^2}{16} \frac{4}{(4+\lambda^2)^2} \cos \frac{2\pi x}{a} + \lambda^2 \left[(a_1 + 2b_1) \cosh \frac{\pi x}{a} + b_1 \left(\frac{\pi x}{a} \right) \sinh \frac{\pi x}{a} \right] \right\} \cos \frac{\pi y}{b} \right. \\ &+ \left. \left\{ \frac{f \lambda^2}{16} \frac{1}{(1+4\lambda^2)^2} \cos \frac{\pi x}{a} + 4\lambda^2 \left[(a_2 + 2b_2) \cosh \frac{2\pi x}{a} + b_2 \left(\frac{2\pi x}{a} \right) \sinh \frac{2\pi x}{a} \right] \right\} \cos \frac{2\pi y}{b} \right] \end{aligned}$$

$$\begin{aligned} \tilde{G}_{\lambda=b} &= E \left(\frac{a}{R} \right)^2 \frac{f \lambda^2}{g} \left[\frac{f \lambda^2}{16} \cos \frac{\pi x}{a} + \frac{f \lambda^2}{64} \cos \frac{2\pi x}{a} + 2a_0 \right. \\ &- \left. \left\{ \left(\frac{f \lambda^2}{g} - 1 \right) \frac{\cos \frac{\pi x}{a}}{(1+\lambda^2)^2} + \frac{f \lambda^2}{16} \frac{4}{(4+\lambda^2)^2} \cos \frac{2\pi x}{a} + \lambda^2 \left[(a_1 + 2b_1) \cosh \frac{\pi x}{a} + b_1 \left(\frac{\pi x}{a} \right) \sinh \frac{\pi x}{a} \right] \right\} \right. \\ &+ \left. \left\{ \frac{f \lambda^2}{16} \frac{1}{(1+4\lambda^2)^2} \cos \frac{\pi x}{a} + 4\lambda^2 \left[(a_2 + 2b_2) \cosh \frac{2\pi x}{a} + b_2 \left(\frac{2\pi x}{a} \right) \sinh \frac{2\pi x}{a} \right] \right\} \right] \end{aligned}$$

$$-\frac{\sigma}{E} = \left(\frac{a}{R}\right)^2 \frac{h^2}{8} \left[2a_0 - \frac{\lambda}{\pi} \left\{ (a_1 + b_1) \sin h \lambda \pi + b_1 (\pi \lambda) \cosh \pi \lambda \right\} \right. \\ \left. + \frac{2\lambda}{\pi} \left\{ (a_2 + b_2) \sin h 2\pi + b_2 (2\pi \lambda) \cosh 2\pi \lambda \right\} \right] = \underline{\underline{\left(\frac{a}{R}\right)^2 \frac{h^2}{8} [2a_0] - \frac{\sigma}{E}}}$$

$$\frac{1}{2} \left(\frac{a}{R}\right)^2 = \left(\frac{a}{R}\right)^2 \frac{h^2}{8} \left[\frac{h^2}{64} \left(3 + 4 \cos \frac{\pi x}{a} + \cos \frac{2\pi x}{a} \right) \left(1 - \cos \frac{2\pi y}{b} \right) \right] \\ = \left(\frac{a}{R}\right)^2 \frac{h^2}{8} \left[\frac{h^2}{64} \left(3 + 4 \cos \frac{\pi x}{a} + \cos \frac{2\pi x}{a} \right) \right. \\ \left. - \frac{h^2}{64} \left(3 + 4 \cos \frac{\pi x}{a} + \cos \frac{2\pi x}{a} \right) \cos \frac{2\pi y}{b} \right]$$

$$\frac{\partial v}{\partial y} = \left(\frac{a}{R}\right)^2 \frac{h^2}{8} \left[-\frac{3h^2}{64} + 2a_0 \right. \\ \left. + \left\{ \left(\frac{h^2}{8} - 1\right) \frac{\cos \frac{\pi x}{a}}{(1+h^2)^2} + \frac{h^2}{16} \frac{4}{(4+h^2)^2} \cos \frac{2\pi x}{a} + \dots \dots \dots \right\} \cos \frac{\pi y}{b} \right. \\ \left. + \left\{ \dots \dots \dots \right\} \cos \frac{2\pi y}{b} \right]$$

$$\frac{v}{b} = \left(\frac{a}{R}\right)^2 \frac{h^2}{8} \left[-\frac{3}{64} (h^2) + 2a_0 \right] = - \left(\frac{a}{R}\right)^2 \frac{h^2}{8} \frac{3}{64} (h^2) - \frac{\sigma}{E}$$

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$$\frac{\Delta \rho}{E_{abl}} = -(\frac{\sigma^2}{E}) - (\frac{Q}{R})(\frac{A}{f})^2 (\frac{3}{f} \pi \lambda^2) \frac{\sigma}{E}$$

$$\begin{aligned} \frac{1}{g_{ab}} \int_0^1 \int_0^1 \frac{Q^2}{(E)} \lambda^2 dy &= (\frac{Q}{R})^4 (\frac{A}{f})^2 \left[\frac{1}{4} (\frac{A^2}{16} - 1)^2 + \frac{1}{f} \left\{ (\frac{A^2}{4} - 1) \frac{A^4}{(1+\lambda^2)^2} \right\} + \frac{1}{f} \left\{ \frac{A^2}{16} \frac{A^4}{(4+\lambda^2)^2} \right\} \right]^2 \\ &- \frac{1}{2} \lambda^4 (\frac{A^2}{16} - 1) \int_0^1 \left\{ a_1 \cosh \frac{\pi \lambda x}{a} + b_1 (\frac{\pi \lambda x}{a}) \sinh \frac{\pi \lambda x}{a} \right\} d(\frac{x}{a}) \\ &- \frac{1}{2} \lambda^4 (\frac{A^2}{f} - 1) \frac{\lambda^4}{(1+\lambda^2)^2} \int_0^1 \left\{ a_1 \cosh \frac{\pi \lambda x}{a} + b_1 (\frac{\pi \lambda x}{a}) \sinh \frac{\pi \lambda x}{a} \right\} \cos \frac{\pi x}{a} d(\frac{x}{a}) \\ &- \frac{1}{2} \lambda^4 \frac{A^2}{16} \frac{\lambda^4}{(4+\lambda^2)^2} \int_0^1 \left\{ a_1 \cosh \frac{\pi \lambda x}{a} + b_1 (\frac{\pi \lambda x}{a}) \sinh \frac{\pi \lambda x}{a} \right\} \cos \frac{2\pi x}{a} d(\frac{x}{a}) \\ &+ \frac{1}{4} \lambda^8 \int_0^1 \left\{ a_1 \cosh \frac{\pi \lambda x}{a} + b_1 (\frac{\pi \lambda x}{a}) \sinh \frac{\pi \lambda x}{a} \right\} \left[\frac{1}{4} d(\frac{x}{a})^2 + \frac{1}{f} \left\{ \frac{A^2}{16} \frac{4\lambda^4}{(1+\lambda^2)^2} \right\} \right]^2 \\ &- 2\lambda^4 \frac{A^2}{64} \int_0^1 \left\{ a_2 \cosh \frac{2\pi \lambda x}{a} + b_2 (\frac{2\pi \lambda x}{a}) \sinh \frac{2\pi \lambda x}{a} \right\} d(\frac{x}{a}) \\ &- 2\lambda^4 \frac{A^2}{16} \frac{4\lambda^4}{(1+4\lambda^2)^2} \int_0^1 \left\{ a_2 \cosh \frac{2\pi \lambda x}{a} + b_2 (\frac{2\pi \lambda x}{a}) \sinh \frac{2\pi \lambda x}{a} \right\} \cos \frac{2\pi x}{a} d(\frac{x}{a}) \\ &+ 4\lambda^8 \int_0^1 \left\{ a_2 \cosh \frac{2\pi \lambda x}{a} + b_2 (\frac{2\pi \lambda x}{a}) \sinh \frac{2\pi \lambda x}{a} \right\} \left[\frac{1}{4} d(\frac{x}{a})^2 \right] \end{aligned}$$

$$\begin{aligned}
\frac{1}{2ab} \int_0^a \int_0^b \left(\frac{\sigma_y^2}{E} \right)^2 dx dy &= \left(\frac{a^4}{R} \right) \left(\frac{1}{f} \right)^2 \left[\frac{1}{4} \left(\frac{f\pi^2}{16} \lambda^2 \right)^2 + \frac{1}{4} \left(\frac{f\pi^2}{64} \lambda^2 \right)^2 + \frac{1}{f} \left(\frac{f\pi^2}{8} - 1 \right) \frac{\lambda^2}{(1+\lambda^2)^2} \right] \left\{ \frac{1}{f} \left(\frac{f\pi^2}{16} \frac{4\lambda^2}{(4+\lambda^2)^2} \right) \right\}^2 \\
&+ \frac{1}{2} \lambda^4 \left(\frac{f\pi^2}{8} - 1 \right) \frac{\lambda^2}{(1+\lambda^2)^2} \int_0^1 \left[(a_1 + 2b_1) \cosh \frac{\pi \lambda x}{a} + b_1 \left(\frac{\pi \lambda x}{a} \right) \sinh \frac{\pi \lambda x}{a} \right] \cos \frac{\pi x}{a} d\left(\frac{x}{a} \right) \\
&+ \frac{1}{2} \lambda^4 \frac{f\pi^2}{16} \frac{4\lambda^2}{(4+\lambda^2)^2} \int_0^1 \left\{ (a_1 + 2b_1) \cosh \frac{\pi \lambda x}{a} + b_1 \left(\frac{\pi \lambda x}{a} \right) \sinh \frac{\pi \lambda x}{a} \right\} \cos \frac{2\pi x}{a} d\left(\frac{x}{a} \right) \\
&+ \frac{1}{4} \lambda^8 \int_0^1 \left\{ (a_1 + 2b_1) \cosh \frac{\pi \lambda x}{a} + b_1 \left(\frac{\pi \lambda x}{a} \right) \sinh \frac{\pi \lambda x}{a} \right\}^2 d\left(\frac{x}{a} \right) + \frac{1}{f} \left\{ \frac{f\pi^2}{16} \frac{\lambda^2}{(1+4\lambda^2)^2} \right\}^2 \\
&+ 2\lambda^4 \frac{f\pi^2}{16} \frac{\lambda^2}{(1+4\lambda^2)^2} \int_0^1 \left\{ (a_2 + 2b_2) \cosh \frac{2\pi \lambda x}{a} + b_2 \left(\frac{2\pi \lambda x}{a} \right) \sinh \frac{2\pi \lambda x}{a} \right\} \cos \frac{\pi x}{a} d\left(\frac{x}{a} \right) \\
&+ 4\lambda^8 \int_0^1 \left\{ (a_2 + 2b_2) \cosh \frac{2\pi \lambda x}{a} + b_2 \left(\frac{2\pi \lambda x}{a} \right) \sinh \frac{2\pi \lambda x}{a} \right\}^2 d\left(\frac{x}{a} \right) + \frac{1}{2} \left(\frac{\sigma}{E} \right)^2
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{a_0} \int_0^a \int_0^b \frac{v_{xy}^2}{(E)} dx dy = \left(\frac{a}{R} \right)^4 \left(\frac{f}{\delta} \right)^2 \left[\frac{f}{4} \left(\frac{f^2}{\delta^2} - 1 \right) \frac{\lambda^3}{(1+\lambda^2)^2} \right]^2 + \frac{1}{4} \left\{ \frac{f^2}{16} \frac{2\lambda^3}{(4+\lambda^2)^2} \right\}^2 \\
& + \lambda^4 \left(\frac{f^2}{\delta^2} - 1 \right) \frac{\lambda^3}{(1+\lambda^2)^2} \int_0^1 \left\{ (a_1 + b_1) \sin \frac{\pi \lambda^X}{a} + b_1 \left(\frac{\pi \lambda^X}{a} \right) \cosh \frac{\pi \lambda^X}{a} \right\} \sin \frac{\pi^X}{a} d \left(\frac{X}{a} \right) \\
& + \lambda^4 \left(\frac{f^2}{16} \right) \frac{2\lambda^3}{(4+\lambda^2)^2} \int_0^1 \left\{ (a_1 + b_1) \sin \frac{\pi \lambda^X}{a} + b_1 \left(\frac{\pi \lambda^X}{a} \right) \cosh \frac{\pi \lambda^X}{a} \right\} \sin \frac{2\pi^X}{a} d \left(\frac{X}{a} \right) \\
& + \frac{1}{2} \lambda^8 \int_0^1 \left\{ (a_1 + b_1) \sin \frac{\pi \lambda^X}{a} + b_1 \left(\frac{\pi \lambda^X}{a} \right) \cosh \frac{\pi \lambda^X}{a} \right\}^2 d \left(\frac{X}{a} \right) + \frac{1}{4} \left\{ \frac{f^2}{16} \frac{2\lambda^3}{(4+\lambda^2)^2} \right\}^2 \\
& + 4\lambda^4 \frac{f^2}{16} \frac{2\lambda^3}{(1+4\lambda^2)^2} \int_0^1 \left\{ (a_2 + b_2) \sin \frac{2\pi \lambda^X}{a} + b_2 \left(\frac{2\pi \lambda^X}{a} \right) \cosh \frac{2\pi \lambda^X}{a} \right\} \sin \frac{\pi^X}{a} d \left(\frac{X}{a} \right) \\
& + 8\lambda^8 \int_0^1 \left\{ (a_2 + b_2) \sin \frac{2\pi \lambda^X}{a} + b_2 \left(\frac{2\pi \lambda^X}{a} \right) \cosh \frac{2\pi \lambda^X}{a} \right\}^2 d \left(\frac{X}{a} \right) \Big]
\end{aligned}$$

$$\int_0^1 \frac{\cos \frac{2\pi x}{a} \cosh \frac{\pi \lambda x}{a} d(\frac{x}{a})}{\lambda + 2i} = \frac{1}{2\pi} \int_0^\pi [\cosh(\lambda + 2i)\theta + \cosh(\lambda - 2i)\theta] d\theta$$

$$= \frac{1}{2\pi} \left[\frac{\sinh(\lambda + 2i)\pi}{\lambda + 2i} + \frac{\sinh(\lambda - 2i)\pi}{\lambda - 2i} \right] = \frac{\lambda}{\pi} \frac{\sinh \lambda \pi}{4 + \lambda^2}$$

$$\int_0^1 \frac{\cos \frac{2\pi x}{a} (\frac{\pi \lambda x}{a}) \sinh \frac{\pi \lambda x}{a} d(\frac{x}{a})}{(\lambda + 2i)^2} = \frac{\lambda}{2\pi} \left[\pi \left\{ \frac{\cosh(\lambda + 2i)\pi}{\lambda + 2i} + \frac{\cosh(\lambda - 2i)\pi}{\lambda - 2i} \right\} \right.$$

$$\left. - \left\{ \frac{\sinh(\lambda + 2i)\pi}{(\lambda + 2i)^2} + \frac{\sinh(\lambda - 2i)\pi}{(\lambda - 2i)^2} \right\} \right] = \left[\frac{\lambda^2 \cosh \lambda \pi}{(4 + \lambda^2)} + \frac{\lambda(4 - \lambda^2) \sinh \lambda \pi}{\pi(4 + \lambda^2)^2} \right]$$

$$\begin{aligned}
& \frac{1}{2ab} \int_0^b \int_0^a \left(\frac{\partial x}{\partial y} \right)^2 dx dy = \left(\frac{a}{b} \right)^4 \left(\frac{1}{8} \right)^2 \left[\frac{1}{4} \left(\frac{\pi^2}{16} - 1 \right)^2 + \frac{1}{8} \left(\left| \frac{-\pi^2}{8} - 1 \right| \right)^2 + \frac{1}{8} \left(\frac{\pi^2}{16} \frac{1^4}{(4\lambda^2)^2} \right)^2 \right] \\
& - \frac{1^4}{2} \left(\frac{\pi^2}{16} - 1 \right) \left[(a_1 - b_1) \frac{\sinh \pi \lambda}{\pi \lambda} + b_1 \cosh \pi \lambda \right] \\
& + \frac{1^4}{2} \left(\frac{\pi^2}{8} - 1 \right) \frac{1^4}{(1+\lambda^2)^2} \left\{ a_1 \frac{1}{\pi} \frac{\sinh \lambda \pi}{1+\lambda^2} + \left[\frac{\lambda^2 \cosh \lambda \pi}{1+\lambda^2} + \frac{\lambda(1-\lambda^2) \sinh \lambda \pi}{\pi(1+\lambda^2)^2} \right] b_1 \right\} \\
& - \frac{1^4}{2} \frac{\pi^2}{16} \frac{1^4}{(4+\lambda^2)^2} \left\{ \frac{1}{\pi} \frac{\sinh \lambda \pi}{4+\lambda^2} a_1 + \left[\frac{\lambda^2 \cosh \lambda \pi}{4+\lambda^2} + \frac{\lambda(4-\lambda^2) \sinh \lambda \pi}{\pi(4+\lambda^2)^2} \right] b_1 \right\} \\
& + \frac{1^8}{4} \left\{ a_1^2 \left(\frac{1}{2} + \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right) + \frac{a_1 b_1}{2} \left(\cosh 2\pi \lambda - \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right) + b_1^2 \left(-\frac{\pi^2}{6} + \frac{1}{8} \left[(4\pi^2 \lambda^2 + 2) \frac{\sinh 2\pi \lambda}{2\pi \lambda} - 2 \cosh 2\pi \lambda \right] \right) \right\} \\
& + \frac{1}{4} \left(\frac{\pi^2}{64} \right)^2 + \frac{1}{8} \left\{ \frac{\pi^2}{16} \frac{4\lambda^4}{(1+4\lambda^2)^2} \right\}^2 - 2\lambda^4 \frac{\pi^2}{64} \left\{ (a_2 - b_2) \frac{\sinh 2\pi \lambda}{2\pi \lambda} + b_2 \cosh 2\pi \lambda \right\} \\
& + 2\lambda^4 \frac{\pi^2}{16} \frac{4\lambda^4}{(1+4\lambda^2)^2} \left\{ \frac{2\lambda}{\pi} \frac{\sinh 2\lambda \pi}{(1+4\lambda^2)} a_2 + \left[\frac{4\lambda^2 \cosh 2\lambda \pi}{1+4\lambda^2} + \frac{2\lambda(1-4\lambda^2) \sinh 2\lambda \pi}{\pi(1+4\lambda^2)^2} \right] b_1 \right\} \\
& + 4\lambda^8 \left\{ a_2^2 \left(\frac{1}{2} + \frac{\sinh 4\pi \lambda}{4\pi \lambda} \right) + \frac{a_2 b_2}{2} \left(\cosh 4\pi \lambda - \frac{\sinh 4\pi \lambda}{4\pi \lambda} \right) + b_2^2 \left(-\frac{4\pi^2}{6} + \frac{1}{8} \left[(16\pi^2 \lambda^2 + 2) \frac{\sinh 4\pi \lambda}{4\pi \lambda} - 2 \cosh 4\pi \lambda \right] \right) \right\}
\end{aligned}$$

$$\cosh \pi \lambda \cdot a_1 + \pi \lambda \sinh \pi \lambda \cdot b_1 = \frac{1}{\lambda^4} \left[\left(\frac{f\pi^2}{16} - 1 \right) - \left(\frac{f\pi^2}{8} - 1 \right) \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{f\pi^2}{16} \frac{\lambda^4}{(4+\lambda^2)^2} \right] \frac{542}{19}$$

$$\sinh \pi \lambda \cdot a_1 + (\sinh \pi \lambda + \pi \lambda \cosh \pi \lambda) b_1 = 0$$

$$b_1 = \frac{1}{\lambda^4} \left[\left(\frac{f\pi^2}{16} - 1 \right) - \left(\frac{f\pi^2}{8} - 1 \right) \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{f\pi^2}{16} \frac{\lambda^4}{(4+\lambda^2)^2} \right] \left[- \frac{\frac{\sinh \pi \lambda}{\pi \lambda}}{1 + \frac{\sinh 2\pi \lambda}{2\pi \lambda}} \right]$$

$$a_1 = \frac{1}{\lambda^4} \left[\left(\frac{f\pi^2}{16} - 1 \right) - \left(\frac{f\pi^2}{8} - 1 \right) \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{f\pi^2}{16} \frac{\lambda^4}{(4+\lambda^2)^2} \right] \left[+ \frac{\cosh \pi \lambda + \frac{\sinh \pi \lambda}{\pi \lambda}}{1 + \frac{\sinh 2\pi \lambda}{2\pi \lambda}} \right]$$

$$a_2 = \frac{1}{16\lambda^4} \left(\frac{f\pi^2}{16} \right) \left\{ 1 - \frac{16\lambda^4}{(1+4\lambda^2)^2} \right\} \left\{ \frac{\cosh 2\pi \lambda + \frac{\sinh 2\pi \lambda}{2\pi \lambda}}{1 + \frac{\sinh 4\pi \lambda}{4\pi \lambda}} \right\}$$

$$b_2 = \frac{1}{16\lambda^4} \left(\frac{f\pi^2}{16} \right) \left\{ 1 - \frac{16\lambda^4}{(1+4\lambda^2)^2} \right\} \left\{ - \frac{\frac{\sinh 2\pi \lambda}{2\pi \lambda}}{1 + \frac{\sinh 4\pi \lambda}{4\pi \lambda}} \right\}$$

$$\begin{aligned}
\frac{1}{g_{ab}} \int_0^b \left(\frac{\partial y}{\partial x} \right)^2 dx dy &= \left(\frac{g}{R} \right)^2 \left(\frac{1}{f} \right)^2 \left[\frac{1}{4} \left(\frac{f}{16} \lambda^2 \right)^2 + \frac{1}{4} \left(\frac{f}{64} \lambda^2 \right)^2 + \frac{1}{8} \left(\frac{f}{8} \lambda^2 \right)^2 + \frac{1}{8} \left(\frac{f}{16} \lambda^2 \right)^2 \right] \left\{ \frac{\lambda^2}{(1+\lambda^2)^2} + \frac{1}{8} \left(\frac{f}{8} \lambda^2 \right)^2 + \frac{1}{8} \left(\frac{f}{16} \lambda^2 \right)^2 \right\}^2 \\
&- \frac{\lambda^4}{2} \left(\frac{f}{8} \lambda^2 - 1 \right) \frac{\lambda^2}{(1+\lambda^2)^2} \left\{ (a_1 + 2b_1) \frac{1}{\pi} \frac{\sinh \lambda \pi}{(1+\lambda^2)} + \left[\frac{\lambda^2 \cosh \lambda \pi}{1+\lambda^2} + \frac{\lambda(1-\lambda^2) \sinh \lambda \pi}{\pi(1+\lambda^2)^2} \right] b_1 \right\} \\
&+ \frac{\lambda^4}{2} \frac{f}{16} \frac{\lambda^2}{(4+\lambda^2)^2} \left\{ (a_1 + 2b_1) \frac{1}{\pi} \frac{\sinh \lambda \pi}{4+\lambda^2} + \left[\frac{\lambda^2 \cosh \lambda \pi}{4+\lambda^2} + \frac{\lambda(4-\lambda^2) \sinh \lambda \pi}{\pi(4+\lambda^2)^2} \right] b_1 \right\} \\
&+ \frac{\lambda^8}{4} \left\{ (a_1 + 2b_1)^2 \left(\frac{1}{2} + \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right) + \frac{(a_1 + 2b_1)b_1}{2} \left(\cosh 2\pi \lambda - \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right) + b_1^2 \left(-\frac{\pi \lambda^2}{6} + \frac{1}{8} \left[(4\pi \lambda^2 + 2) \frac{\sinh 2\pi \lambda}{2\pi \lambda} - 2 \cosh 2\pi \lambda \right] \right) \right\} \\
&+ \frac{1}{f} \left\{ \frac{f}{16} \frac{\lambda^2}{(1+\lambda^2)^2} \right\}^2 - 2\lambda^4 \frac{f}{16} \frac{\lambda^2}{(1+\lambda^2)^2} \left\{ (a_2 + 2b_2) \frac{2\lambda}{\pi} \frac{\sinh 2\pi \lambda}{(1+\lambda^2)} + \left[\frac{4\lambda^2 \cosh 2\pi \lambda}{1+\lambda^2} + \frac{2\lambda(1-\lambda^2) \sinh 2\pi \lambda}{\pi(1+\lambda^2)^2} \right] b_2 \right\} \\
&+ 4\lambda^8 \left\{ (a_2 + 2b_2)^2 \left(\frac{1}{2} + \frac{\sinh 4\pi \lambda}{4\pi \lambda} \right) + \frac{b_2(a_2 + 2b_2)}{2} \left(\cosh 4\pi \lambda - \frac{\sinh 4\pi \lambda}{4\pi \lambda} \right) + b_2^2 \left(-\frac{4\pi \lambda^2}{6} + \frac{1}{8} \left[(16\pi \lambda^2 + 2) \frac{\sinh 4\pi \lambda}{4\pi \lambda} - 2 \cosh 4\pi \lambda \right] \right) \right\} \\
&+ \frac{1}{2} \left(\frac{g}{E} \right)^2
\end{aligned}$$

$$\begin{aligned}
\frac{1}{ab} \int_0^a \int_0^b \left(\frac{xy}{E} \right)^2 dx dy &= \left(\frac{a}{R} \right)^4 \left(\frac{b}{\rho} \right)^2 \left[\frac{1}{4} \left(\frac{b^2}{\rho^2} - 1 \right) \frac{b^3}{(1+\lambda^2)^2} \right]^2 + \frac{1}{4} \left\{ \frac{\rho^2}{16} \frac{2b^3}{(4+\lambda^2)^2} \right\}^2 \\
&+ \lambda^4 \left(\frac{b^2}{\rho^2} - 1 \right) \frac{b^3}{(1+\lambda^2)^2} \left\{ (a_1+b_1) \frac{1}{\pi} \frac{\sinh \lambda \pi}{(1+\lambda^2)} + \left[\frac{\lambda \cosh \lambda \pi}{1+\lambda^2} - \frac{2\lambda^2 \sinh \lambda \pi}{\pi(1+\lambda^2)^2} \right] b_1 \right\} \\
&- \lambda^4 \left(\frac{b^2}{16} \right) \frac{2b^3}{(4+\lambda^2)^2} \left\{ -(a_1+b_1) \frac{2}{\pi} \frac{\sinh \lambda \pi}{(4+\lambda^2)} + \left[\frac{2\lambda \cosh \lambda \pi}{4+\lambda^2} - \frac{4\lambda^2 \sinh \lambda \pi}{\pi(4+\lambda^2)^2} \right] b_1 \right\} \\
&+ \frac{\lambda^6}{2} \left\{ (a_1+b_1)^2 \left(-\frac{1}{2} + \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right) + \frac{b_1(a_1+b_1)}{2} \left(\cosh 2\pi \lambda - \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right) + b_1^2 \left(\frac{\pi^2 \lambda^2}{6} + \frac{1}{\rho^2} \left[(4\pi^2 \lambda^2) \frac{\sinh 2\pi \lambda}{2\pi \lambda} - 2 \cosh 2\pi \lambda \right] \right) \right\} \\
&+ \frac{1}{4} \left\{ \frac{\rho^2}{16} \frac{2b^3}{(1+4\lambda^2)^2} \right\}^2 + 4\lambda^4 \frac{\rho^2}{16} \frac{2b^3}{(1+4\lambda^2)^2} \left\{ (a_2+b_2) \frac{1}{\pi} \frac{\sinh 2\pi \lambda}{(1+4\lambda^2)} + \left[\frac{2\lambda \cosh 2\pi \lambda}{1+4\lambda^2} - \frac{\rho^2 \lambda^2 \sinh 2\pi \lambda}{\pi(1+4\lambda^2)^2} \right] b_2 \right\} \\
&+ \rho \lambda^6 \left\{ (a_2+b_2)^2 \left(-\frac{1}{2} + \frac{\sinh 4\pi \lambda}{4\pi \lambda} \right) + \frac{b_2(a_2+b_2)}{2} \left(\cosh 4\pi \lambda - \frac{\sinh 4\pi \lambda}{4\pi \lambda} \right) + \right. \\
&\quad \left. + b_2^2 \left(\frac{4\pi^2 \lambda^2}{6} + \frac{1}{\rho^2} \left[(16\pi^2 \lambda^2 + 2) \frac{\sinh 4\pi \lambda}{4\pi \lambda} - 2 \cosh 4\pi \lambda \right] \right) \right\}
\end{aligned}$$

Terms independent upon the a_i 's & b_i 's

$$\frac{1}{4} \left(\frac{f^2}{16} - 1 \right)^2 + \lambda^4 \frac{17}{16 \times 1024} \left(\frac{f^2}{16} \right)^2 + \frac{\lambda^4}{(1+\lambda^2)^2} \frac{1}{8} \left(\frac{f^2}{8} - 1 \right)^2 + \frac{\lambda^4}{(4+\lambda^2)^2} \frac{1}{8} \left(\frac{f^2}{16} \right)^2 + \frac{1}{4} \left(\frac{f^2}{64} \right)^2 + \frac{\lambda^4}{(1+4\lambda^2)^2} \frac{1}{8} \left(\frac{f^2}{16} \right)^2$$

Terms linear in a_i, b_i .

$$\begin{aligned} & -\frac{\lambda^4}{2} \left(\frac{f^2}{16} - 1 \right) \left\{ (a_1 - b_1) \frac{\sinh \lambda \pi}{\pi \lambda} + b_1 \cosh \lambda \pi \right\} + \frac{\lambda^4}{2} \left(\frac{f^2}{8} - 1 \right) \frac{\lambda^3}{(1+\lambda^2)^3} \frac{\sinh \lambda \pi}{\pi} \left\{ \lambda^2 a_1 - (a_1 + 2b_1) \right. \\ & \quad \left. + 2(a_1 + b_1) \right\} \\ & + \frac{\lambda^4}{2} \left(\frac{f^2}{8} - 1 \right) \frac{\lambda^4}{(1+\lambda^2)^3} \cosh \lambda \pi \left\{ \lambda^2 b_1 - b_1 + 2b_1 \right\} + \frac{\lambda^4}{2} \left(\frac{f^2}{8} - 1 \right) \frac{\lambda^3}{(1+\lambda^2)^4} \frac{\sinh \lambda \pi}{\pi} \left\{ \lambda^2 (1-\lambda^2) b_1 - (1-\lambda^2) b_1 \right. \\ & \quad \left. - 4\lambda^2 b_1 \right\} \end{aligned}$$

$$-\frac{\lambda^4}{2} \frac{f^2}{16} \frac{\lambda^3}{(4+\lambda^2)^3} \frac{\sinh \lambda \pi}{\pi} \left\{ \lambda^2 a_1 - 4(a_1 + 2b_1) + 8(a_1 + b_1) \right\}$$

$$-\frac{\lambda^4}{2} \frac{f^2}{16} \frac{\lambda^4}{(4+\lambda^2)^3} \cosh \lambda \pi \left\{ \lambda^2 b_1 - 4b_1 + 8b_1 \right\} - \frac{\lambda^4}{2} \frac{f^2}{16} \frac{\lambda^3}{(4+\lambda^2)^4} \frac{\sinh \lambda \pi}{\pi} \left\{ \lambda^2 (4-\lambda^2) b_1 - 4(4-\lambda^2) b_1 \right. \\ \quad \left. - 16\lambda^2 b_1 \right\}$$

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$$= \frac{\lambda^4}{2} \left[- \left(\frac{\pi^2}{16} - 1 \right) (a_1 - b_1) \frac{\sinh \pi \lambda}{\pi \lambda} + b_1 \cosh \pi \lambda \right] + \left(\frac{\pi^2}{8} - 1 \right) \frac{\lambda^4}{(1 + \lambda^2)^2} \left\{ \frac{\sinh \lambda \pi}{\lambda \pi} (a_1 - b_1) + b_1 \cosh \pi \lambda \right\}$$

$$- \frac{\pi^2}{16} \frac{\lambda^4}{(4 + \lambda^2)^2} \left\{ \frac{\sinh \lambda \pi}{\lambda \pi} (a_1 - b_1) + b_1 \cosh \pi \lambda \right\}$$

The terms linear in a_2, b_2

$$2\lambda^4 \left[- \frac{\pi^2}{64} \left\{ (a_2 - b_2) \frac{\sinh 2\pi \lambda}{2\pi \lambda} + \frac{1}{2} \cosh 2\pi \lambda \right\} + \frac{\pi^2}{16} \frac{\lambda^4}{(1 + \lambda^2)^2} \left\{ \frac{\sinh 2\pi \lambda}{2\pi \lambda} (a_2 - b_2) + b_2 \cosh 2\pi \lambda \right\} \right]$$

The terms linear in a_1, b_1

$$- \left[\left(\frac{\pi^2}{16} - 1 \right) - \left(\frac{\pi^2}{8} - 1 \right) \frac{\lambda^4}{(1 + \lambda^2)^2} + \frac{\pi^2}{16} \frac{\lambda^4}{(4 + \lambda^2)^2} \right]^2 \frac{\left(\frac{\sinh \pi \lambda}{\pi \lambda} \right)^2}{1 + \frac{\sinh 2\pi \lambda}{2\pi \lambda}}$$

The terms linear in a_2, b_2

$$- \frac{1}{16} \left[\frac{\pi^2}{16} \left\{ 1 - \frac{16\lambda^4}{(1 + 4\lambda^2)^2} \right\} \right]^2 \frac{\left(\frac{\sinh 2\pi \lambda}{2\pi \lambda} \right)^2}{1 + \frac{\sinh 4\pi \lambda}{4\pi \lambda}}$$

Terms of containing

$a_1, b_1,$

$$\frac{1}{4} \left[\left(\frac{-\lambda^2}{16} - 1 \right) - \left(\frac{-\lambda^2}{8} - 1 \right) \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{-\lambda^2 - \lambda^4}{16} \frac{\lambda^4}{(4+\lambda^2)^2} \right]^2 \frac{\cosh \pi \lambda}{\pi \lambda} \left\{ (2\pi^2 \lambda^2 - 1) \frac{\sinh 2\pi \lambda}{\pi \lambda} + 2 \left(\cosh \pi \lambda - \frac{\sinh 2\pi \lambda}{\pi \lambda} \right) \right\} \frac{(1 + \frac{\sinh 2\pi \lambda}{2\pi \lambda})^2}{(1 + \frac{\sinh 2\pi \lambda}{2\pi \lambda})^2}$$

Terms of containing

a_2, b_2

$$\frac{1}{64} \left[\left(\frac{-\lambda^2}{16} \right)^2 \left\{ 1 - \frac{16\lambda^4}{(1+\lambda^2)^2} \right\}^2 \right] \frac{\sinh 2\pi \lambda}{2\pi \lambda} \left\{ (8\pi^2 \lambda^2 - 1) \frac{\sinh 2\pi \lambda}{2\pi \lambda} + 2 \left(\cosh 2\pi \lambda - \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right) \right\} \frac{(1 + \frac{\sinh 2\pi \lambda}{4\pi \lambda})^2}{(1 + \frac{\sinh 2\pi \lambda}{4\pi \lambda})^2}$$

$$H_1(\lambda) = \frac{17}{16384} (1 + \lambda^4) + \frac{1}{512} \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{1}{2048} \frac{\lambda^4}{(4+\lambda^2)^2} + \frac{1}{2048} \frac{\lambda^4}{(1+4\lambda^2)^2} + \frac{1}{1024} \left[1 - 2 \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{\lambda^4}{(4+\lambda^2)^2} \right]^2 \frac{\sinh \pi \lambda}{\pi \lambda} \left\{ (2\pi^2 \lambda^2 - 1) \frac{\sinh 2\pi \lambda}{2\pi \lambda} + 2 \left(\cosh \pi \lambda - \frac{\sinh 2\pi \lambda}{\pi \lambda} \right) \right\} \frac{(1 + \frac{\sinh 2\pi \lambda}{2\pi \lambda})^2}{(1 + \frac{\sinh 2\pi \lambda}{2\pi \lambda})^2}$$

$$+ \frac{1}{16384} \left\{ 1 - \frac{16\lambda^4}{(1+\lambda^2)^2} \right\}^2 \frac{\sinh 2\pi \lambda}{2\pi \lambda} \left\{ (8\pi^2 \lambda^2 - 1) \frac{\sinh 2\pi \lambda}{2\pi \lambda} + 2 \left(\cosh 2\pi \lambda - \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right) \right\} \frac{(1 + \frac{\sinh 2\pi \lambda}{4\pi \lambda})^2}{(1 + \frac{\sinh 2\pi \lambda}{4\pi \lambda})^2}$$

$$H_2(\lambda) = \frac{1}{32} + \frac{1}{32} \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{1}{32} \left[1 - 2 \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{\lambda^4}{(4+\lambda^2)^2} \right] \left[1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right] \\ \frac{\sinh \pi \lambda}{\pi \lambda} \left\{ (2\pi^2 \lambda^2 - 1) \frac{\sinh \pi \lambda}{\pi \lambda} \frac{\sinh 2\pi \lambda}{2\pi \lambda} + 2 (\cosh \pi \lambda - \frac{\sinh \pi \lambda}{\pi \lambda}) \right\} \\ \left(1 + \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right)^2$$

$$H_3(\lambda) = \frac{1}{4} + \frac{1}{8} \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{1}{4} \left[1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right]^2 \frac{\sinh \pi \lambda}{\pi \lambda} \left\{ (2\pi^2 \lambda^2 - 1) \frac{\sinh \pi \lambda}{\pi \lambda} \frac{\sinh 2\pi \lambda}{2\pi \lambda} + 2 (\cosh \pi \lambda - \frac{\sinh \pi \lambda}{\pi \lambda}) \right\} \\ \left(1 + \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right)^2$$

$$\text{Binding energy} = \frac{1}{24} \left(\frac{f}{R} \right)^2 \pi^4 \left\{ \frac{3}{4} + \frac{3}{4} \lambda^4 + \frac{1}{2} \lambda^2 \right\} = \left(\frac{f}{R} \right)^2 \frac{1}{8} \pi^4 \left\{ \frac{1}{32} (1+\lambda^4) + \frac{1}{48} \lambda^2 \right\}$$

$$\lambda^2 K = 2 \left[\frac{512}{9} H_1 H_3 \left(\frac{f}{R} \right)^2 + H_3 \left\{ \frac{2}{9} (1+\lambda^4) + \frac{4}{27} \lambda^2 \right\} \right]^{\frac{1}{2}} - 8 H_2 \left(\frac{f}{R} \right)$$

$$f^2 = \pi^2 \left[\frac{8 H_1}{H_3} \left(\frac{f}{R} \right)^2 + \frac{1}{H_3} \left\{ \frac{1}{32} (1+\lambda^4) + \frac{1}{48} \lambda^2 \right\} \right]^{\frac{1}{2}}$$

$$\left(\frac{f}{R} \right) = \pi \left(\frac{f}{R} \right)^{\frac{1}{2}} \left[\frac{8 H_1}{H_3} \left(\frac{f}{R} \right)^{\frac{1}{2}} + \frac{1}{H_3} \left\{ \frac{1}{32} (1+\lambda^4) + \frac{1}{48} \lambda^2 \right\} \right]^{\frac{1}{4}}$$

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λ	$\pi\lambda$	$\ln_{10}(e^{\pi\lambda})$	$e^{\pi\lambda}$	$e^{-\pi\lambda}$	$\sinh \pi\lambda$	$\cosh \pi\lambda$	$\sinh \pi\lambda / \pi\lambda$	$\cosh \pi\lambda$	$\cosh \pi\lambda - \sinh \pi\lambda$
0.05	0.1570746	0.0682188	1.170089	0.85436	0.1577265	1.004118	1.004118	1.012313	0.008245
0.10	0.3141493	0.1364376	1.359108	0.730403	0.3193525	1.016530	1.016530	1.04756	0.033276
0.15	0.4712239	0.2046565	1.601978	0.62428	0.4888750	1.037425	1.037425	1.113103	0.075678
0.20	0.6283185	0.2728753	1.874456	0.533468	0.6706840	1.067108	1.067108	1.20372	0.136664
0.30	0.9424778	0.409329	2.566333	0.389661	1.088360	1.154760	1.154760	1.477977	0.32327
0.40	1.2563371	0.545506	3.512586	0.284610	1.614488	1.284269	1.284269	1.719098	0.434329
0.50	1.5707914	0.6821882	4.810478	0.207880	2.301291	1.465053	1.465053	2.509179	1.044126
0.60	1.8849556	0.818658	6.86665	0.151836	3.217115	1.706232	1.706232	3.368951	1.66219
0.70	2.5132742	1.0915011	12.34529	0.081003	6.132144	2.439903	2.439903	6.213146	3.77243
1.00	3.1415927	1.3643764	23.14070	0.043214	11.54774	3.676078	3.676078	11.59196	7.91588
1.20	3.7699112	1.6372517	43.37623	0.023054	21.62659	5.769894	5.769894	21.69964	15.91975
1.40	4.3982298	1.9101270	81.30683	0.012299	40.64727	9.241734	9.241734	40.65907	31.41734
1.60	5.0265483	2.1830022	152.4060	0.006561	76.19772	15.5945	15.5945	76.20628	61.04683
1.80	5.6548669	2.4557775	265.6785	0.003500	142.8375	25.5921	25.5921	142.8410	122.5818
2.00	6.2831854	2.7267528	535.4917	0.001817	267.7449	42.61293	42.61293	267.7468	225.1339
2.20	6.9115039	3.0016281	1002.756	0.000996	501.8725	72.61480	72.61480	501.8785	479.2637

λ	$e^{2\pi\lambda}$	$e^{-2\pi\lambda}$	$\text{sinh } 2\pi\lambda / 2$	$-\text{sinh } 2\pi\lambda / 2\pi\lambda$	$\cosh 2\pi\lambda$	$\cosh 2\pi\lambda - \text{sinh } 2\pi\lambda$	$\text{sinh } 4\pi\lambda$	$\text{sinh } 4\pi\lambda / 4\pi\lambda$
0.05			0.3193525/2	1.016530	1.049356	0.033226	0.67046640	1.067108
0.10			0.67046640/2	1.067108	1.203722	0.136664	1.614468	1.284469
0.15			1.0883360/2	1.154760	1.477997	0.323237	3.217115	1.706732
0.20			1.614468/2	1.284769	1.719098	0.434329	6.132144	2.439903
0.30			3.217115/2	1.706732	3.368951	1.62219	21.67659	5.749694
0.40			---	2.439903	6.213146	3.77243	76.19942	15.15945
0.50			---	3.676078	11.59196	7.91588	267.7449	42.61293
0.60			---	5.347894	21.69764	15.94975	940.7464	124.7707
0.80			---	15.15945	76.20628	61.04613	11613.79	1155.245
1.00			---	42.61293	267.7468	225.1339	143375.7	11409.474
1.20	1881.4973	0.0005	470.3742	124.7707	940.7489	815.9782	1770016	117377.90
1.40	6110.8006	0.0001	1152.7001	375.7648	3305.400	2929.635	21851340	1242053
1.60	23227.589		5801.8943	1155.245	11613.79	10458.54	269760300	13416770
1.80	81162.205		20403.0513	3608.051	40806.10	37198.05	3330276000	147230500
2.00	286251.36		71187840	11409.474	143375.7	131966.2	41113180000	1635841000
2.20	1002526.1		25188153	36443.81	503263.1	467319.3	507554500000	18329050000

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λ	$\sin 2\pi\lambda$	$\frac{\sin 2\pi\lambda}{1 + \frac{\sin 2\pi\lambda}{2\pi\lambda}}$	$\frac{\sin 2\pi\lambda / 2\pi\lambda}{1 + \frac{\sin 2\pi\lambda}{2\pi\lambda}}$	$\frac{g(\cos \pi\lambda - \frac{\sin 2\pi\lambda}{2\pi\lambda})}{1 + \frac{\sin 2\pi\lambda}{2\pi\lambda}}$	$2\pi^2\lambda^2 - 1$	I
0.05	0.3193525	0.4799435	0.5040966	0.00877	-0.9506520	-0.2355366
0.10	0.6704840	0.4917643	0.5162323	0.032147	-0.8026089	-0.1913133
0.15	1.0183360	0.4814573	0.5359112	0.070243	-0.5558678	-0.1147228
0.20	1.614488	0.4670529	0.5623190	0.11806	-0.2104316	-0.0030193
0.30	3.217115	0.4266252	0.6305508	0.238839	+0.7775248	+0.3431162
0.40	6.132144	0.3734899	0.7072941	0.32524	+2.156273	+0.828891
0.50	11.54874	0.3133041	0.7861456	0.44562	+3.934802	+1.559295
0.60	21.67659	0.2528532	0.8518495	0.492517	+6.106115	+2.369252
0.70	76.19972	0.1509892	0.9381167	0.461901	+11.63309	+4.090907
1.00	267.7649	0.08428719	0.9770710	0.363006	+18.73921	+5.703843
1.20	940.7483	0.04571228	0.9920490	0.25632	+27.42446	+7.163331
1.40	3325.400	0.0245219	0.9973458	0.167274	+37.6485	+8.525184
1.60	11113.79	0.01311093	0.9991351	0.105595	+49.53237	+9.837651
1.80	40604.10	0.00699851	0.9997229	0.067930	+62.95504	+11.12693
2.00	143325.7	0.003736545	0.9999127	0.039461	+77.95684	+12.40511
2.20	503763.1	0.001992459	0.9999726	0.023557	+94.53727	+13.67759

λ	$\frac{\sin(2\pi\lambda)}{1 + \frac{\sin(4\pi\lambda)}{4\pi\lambda}}$	$\frac{\sin(4\pi\lambda)}{1 + \frac{\sin(8\pi\lambda)}{4\pi\lambda}}$	$\frac{2(\cos(2\pi\lambda) - \frac{\sin(4\pi\lambda)}{4\pi\lambda})}{1 + \frac{\sin(8\pi\lambda)}{4\pi\lambda}}$	$\delta\pi\lambda^2 - 1$	II	λ^4	$1+\lambda^2$	$4+\lambda^2$	$1+4\lambda^2$
0.05	0.4917663	0.5162323	0.032147	-0.8806029	-0.1913133	0.0000625	1.0025	4.0025	1+4λ ²
0.10	0.4670529	0.5123190	0.119806	-0.2106317	-0.0030193	0.0001	1.01	4.01	1.040
0.15	0.4266252	0.630508	0.238839	+0.7765288	+0.3431162	0.00050625	1.0225	4.0225	1.090
0.20	0.3734499	0.7092941	0.252524	+2.158223	+0.828891	0.0016	1.04	4.04	1.160
0.30	0.258532	0.8518495	0.492517	+6.106115	+2.36952	0.0081	1.09	4.09	1.360
0.40	0.150892	0.9381167	0.468901	+11.63309	+4.09607	0.0256	1.16	4.16	1.64
0.50	0.08428879	0.977070	0.363006	+18.73921	+5.703843	0.0625	1.25	4.25	2.00
0.60	0.0457128	0.9920490	0.253632	+27.42446	+7.163331	0.1296	1.36	4.36	2.44
0.80	0.01311093	0.9991351	0.105595	+49.53237	+9.837651	0.4096	1.64	4.64	3.56
1.00	0.003734545	0.9999127	0.039461	+77.95684	+12.40511	1.0000	2.00	5.00	5.00
1.20	0.001062973	0.9999915	0.0139033	+112.6978	+14.94676	2.0736	2.44	5.44	6.76
1.40	0.000302550	0.9999992	0.004717	+153.7554	+17.47921	3.8416	2.96	5.96	8.84
1.60	0.000086045	1	0.00159	+201.1295	+20.00672	6.5536	3.56	6.56	11.24
1.80	0.00002450614	1	0.000505	+225.48201	+22.53104	10.4976	4.24	7.24	13.96
2.00	0.00000694684	1	—	+314.8223	+25.05316	16.0000	5.00	8.00	17.00
2.20	0.000001985060	1	—	+381.1511	+27.57367	23.4256	5.84	9.84	20.36

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λ	①	②	③	④	⑤	⑥	⑦	⑧
	$\frac{17}{1634} (1+\lambda^4)$	$\frac{\lambda^4}{(1+\lambda^2)^2}$	$\frac{\lambda^4}{(4+\lambda^2)^2}$	$\frac{\lambda^4}{(1+\lambda^2)^2}$	$\frac{1}{512}$	③+④	$\frac{1}{2048}$	①+⑤+⑦
0.25	0.001037604	0.00000620	0.00000039	0.00000613	0.00000012	0.00000652	0.00000003	0.001037619
0.10	0.001037702	0.00009803	0.00000122	0.00009246	0.00000191	0.00009818	0.000000048	0.001037741
0.15	0.001038123	0.00048422	0.00003129	0.00046610	0.00000966	0.00045739	0.000000223	0.001039292
0.20	0.001039258	0.00147429	0.00009803	0.00118906	0.00000289	0.00128709	0.000000628	0.001042275
0.30	0.001040003	0.00681761	0.00048622	0.00437933	0.00001316	0.00486355	0.000002375	0.001061694
0.40	0.001064616	0.01902497	0.00147929	0.00951814	0.00003758	0.01099743	0.000005370	0.001106689
0.50	0.001102448	0.04000000	0.00346021	0.01562500	0.000078125	0.01908521	0.000009319	0.001189892
0.60	0.001172071	0.07006920	0.00681761	0.021216834	0.000136854	0.024858595	0.000013958	0.001322485
0.70	0.001462598	0.15229030	0.01924977	0.03231915	0.000277442	0.05134412	0.000025070	0.001785110
1.00	0.001075196	0.25000000	0.04000000	0.04000000	0.000488281	0.06000000	0.000039062	0.002602559
1.20	0.003189161	0.34879347	0.07006920	0.04537856	0.00068261	0.11544576	0.000056370	0.003925772
1.40	0.005023634	0.43845873	0.10816828	0.04715952	0.00086365	0.15720780	0.00007610	0.005956809
1.60	0.007837600	0.51710143	0.15229030	0.05182371	0.001009123	0.20416401	0.000097189	0.008947262
1.80	0.011929887	0.58392116	0.20026661	0.05586655	0.001140682	0.25473516	0.000124089	0.013194446
2.00	0.017639166	0.64000000	0.25000000	0.05536332	0.001250000	0.30536332	0.000149103	0.01903827
2.20	0.025043954	0.6827452	0.29976664	0.05511284	0.001332372	0.35627992	0.000173965	0.02685029

λ	⑨ $1-2 \times ③ + ③$	⑩ $1-16 \times ④$	⑪ $\frac{1}{104} ⑨^2 \times I$	⑫ $\frac{1}{1634} ⑩^2 \times II$	⑬ H_1	⑭ $1-⑤$	⑮ $⑨ \times ⑭ \times I$	⑯ H_2
0.05	0.999880	0.9999019	-0.00023011	-0.00000167	+0.00050644	0.9999738	-0.2355323	+0.0238881
0.10	0.9998102	0.9985206	-0.00086358	-0.00000008	+0.000851165	0.9997019	-0.1912582	+0.0257664
0.15	0.9996129	0.9931824	-0.00011204	+0.000002066	+0.00097290	0.9995158	-0.116894	+0.02767738
0.20	0.9971395	0.989750	-0.00002932	+0.000004668	+0.001044711	0.9985207	-0.0030062	+0.03120228
0.30	0.986490	0.92993072	+0.000326319	+0.00012505	+0.00140058	0.9931824	+0.3362954	+0.0497228
0.40	0.9634294	0.8477098	+0.000751341	+0.00017943	+0.001875973	0.9809750	+0.7833850	+0.0532531
0.50	0.9234602	0.7500000	+0.001296984	+0.00019582	+0.002508458	0.960000	+1.3827922	+0.0757226
0.60	0.866672	0.6517066	+0.001739114	+0.00018570	+0.003079567	0.9299308	+1.9095027	+0.0931162
0.70	0.7164444	0.4821831	+0.002039185	+0.00014001	+0.003838296	0.8477097	+2.4776226	+0.1134348
1.00	0.5400000	0.3600000	+0.001624259	+0.000039812	+0.004236610	0.7500000	+2.3100564	+0.112518
1.20	0.3734923	0.2739350	+0.000975388	+0.000004847	+0.004910427	0.6517065	+1.7435605	+0.09662044
1.40	0.2312308	0.2134477	+0.000445138	+0.000004861	+0.00460808	0.5615413	+1.1069576	+0.07954426
1.60	0.1180774	0.1700206	+0.000133945	+0.000001527	+0.004062734	0.4821836	+0.5609313	+0.06493887
1.80	0.0324153	0.1381352	+0.000011417	+0.000001620	+0.01320150	0.4160734	+0.1500705	+0.05418741
2.00	-0.0300000	0.1141819	+0.000010903	+0.000001990	+0.01905116	0.3600000	-0.1337352	+0.04706328
2.20	-0.064804	0.0958195	+0.000055707	+0.000001544	+0.02670254	0.318255	-0.2507366	+0.04379473

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λ	(17) $2 \times 10^2 \times I$	(18) H_3	(19) $\frac{2}{9}(1+\lambda^4) + \frac{4}{27}\lambda^2$	(20) $\frac{29}{\lambda^4}$	(21) $(16) \times (20)$	(22) K_0	(23) H/λ^4	(24) $\frac{512}{9} H_3$
0.05	-0.4416674	+0.1911174	0.2225940	35615.04	6806.54		129.036	1402.880
0.10	-0.3825515	+0.2021933	0.2237259	2237.259	452.3518		85.1165	97.90550
0.15	-0.2297230	+0.2213452	0.2256681	445.7641	98.66774		18.3535	23.11447
0.20	-0.0060207	+0.2494323	0.2285037	142.8148	35.62262		0.6529444	9.265233
0.3	+0.679024	+0.335456	0.2373556	29.30316	9.830202		0.1729035	3.297736
0.4	+1.5953034	+0.4517911	0.2516148	9.828703	4.440520		0.07324020	1.883440
0.5	+2.8250141	+0.6143768	0.2731461	4.370370	2.685054		0.04013523	1.402778
0.6	+4.0777223	+0.7709729	0.3043556	2.348423	1.810573		0.02376209	1.042102
0.8	+5.895477	+1.0059709	0.4080593	0.962385	1.002187		0.009370846	0.536298
1.0	+6.416234	+1.0833529	0.5925925	0.5725925	0.6419868	1.602462	0.004236110	0.2611054
1.2	+6.0844392	+1.0541416	0.8913555	0.4322702	0.4556740	1.350072	0.00236104	0.1449527
1.4	+5.3764695	+0.9768660	1.3612615	0.3556543	0.3474266	1.178858	0.001667745	0.096813
1.6	+4.5860099	+0.8681395	2.0572370	0.31400100	0.2788767	1.056176	0.001385915	0.0700238
1.8	+3.8525226	+0.6045562	3.0350222	0.2891158	0.2326099	0.944592	0.001258105	0.0575857
2.0	+3.2154065	+0.7319256	4.0249360	0.2516835	0.1842136	0.858402	0.001190698	0.0495888
2.2	+2.7632302	+0.6806756	6.1449481	0.2223746	0.1785532	0.845112	0.00114838	0.0444686

$$\sigma_{y=b} = -\tilde{\sigma} + E \left(\frac{q}{R} \right)^2 \frac{\lambda^2}{g} \left[\frac{\lambda^2}{16} \cos \frac{\pi x}{a} + \frac{\lambda^2}{64} \cos \frac{2\pi x}{a} - \left(\frac{\lambda^2}{g^2} - 1 \right) \frac{\cos \frac{\pi x}{a}}{(1+\lambda^2)^2} - \frac{\lambda^2}{16} \frac{\cos \frac{2\pi x}{a}}{(4+\lambda^2)^2} \right]$$

$$- \frac{1}{\lambda^2} \left\{ \left(\frac{\lambda^2}{16} - 1 \right) - \left(\frac{\lambda^2}{g^2} - 1 \right) \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{\lambda^2}{16} \frac{\lambda^4}{(4+\lambda^2)^2} \right\} \left\{ \left(\cosh \pi \lambda - \frac{\sinh 2\pi \lambda}{\pi \lambda} \right) \cosh \frac{\pi x}{a} - \left(\frac{\sinh \pi \lambda}{\pi \lambda} \right) \left(\frac{\pi x}{a} \right) \sinh \frac{\pi x}{a} \right\}$$

$$+ \frac{\lambda^2}{16} \frac{1}{(1+\lambda^2)^2} \cos \frac{\pi x}{a} + \frac{1}{4\lambda^2} \left(\frac{\lambda^2}{16} \right) \left\{ 1 - \frac{16\lambda^4}{(1+\lambda^2)^2} \right\} \left\{ \left(\cosh 2\pi \lambda - \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right) \cosh \frac{2\pi x}{a} - \left(\frac{\sinh 2\pi \lambda}{2\pi \lambda} \right) \left(\frac{2\pi x}{a} \right) \sinh \frac{2\pi x}{a} \right\}$$

$$= -\tilde{\sigma} + E \left(\frac{q}{R} \right)^2 \frac{\lambda^2}{g} \left\{ \left[\frac{\lambda^2}{16} \left\{ 1 - 2 \frac{1}{(1+\lambda^2)^2} + \frac{1}{(1+\lambda^2)^2} \right\} + \frac{1}{(1+\lambda^2)^2} \right] \cos \frac{\pi x}{a} + \right.$$

$$\left. + \frac{\lambda^2}{64} \left\{ 1 - \frac{16}{(4+\lambda^2)^2} \right\} \cos \frac{2\pi x}{a} \right\}$$

$$- \frac{1}{\lambda^2} \left[\frac{\lambda^2}{16} \left\{ 1 - 2 \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{\lambda^4}{(4+\lambda^2)^2} \right\} - \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \right] \left\{ \left(\cosh \pi \lambda - \frac{\sinh \pi \lambda}{\pi \lambda} \right) \cosh \frac{\pi x}{a} - \left(\frac{\sinh \pi \lambda}{\pi \lambda} \right) \left(\frac{\pi x}{a} \right) \sinh \frac{\pi x}{a} \right\}$$

$$+ \frac{1}{\lambda^2} \frac{\lambda^2}{64} \left\{ 1 - \frac{16\lambda^4}{(1+\lambda^2)^2} \right\} \left\{ \left(\cosh 2\pi \lambda - \frac{\sinh 2\pi \lambda}{2\pi \lambda} \right) \cosh \frac{2\pi x}{a} - \left(\frac{\sinh 2\pi \lambda}{2\pi \lambda} \right) \left(\frac{2\pi x}{a} \right) \sinh \frac{2\pi x}{a} \right\}$$

$$\begin{aligned}
 \left(\frac{\phi_y}{E} \right)_{y=b} &= -K + \frac{\lambda^2}{4} \left(\frac{\phi}{E} \right) \left\{ \left[\frac{\pi^2}{8} \frac{\left(\frac{\phi}{E} \right)}{j^2} \left\{ 1 - \frac{2}{(1+\lambda^2)^2} + \frac{1}{(1+\lambda^2)^2} \right\} + \frac{1}{(1+\lambda^2)^2} \right] \cos \frac{\pi x}{a} + \right. \\
 &\quad \left. + \frac{\pi^2}{32} \frac{\left(\frac{\phi}{E} \right)}{j^2} \left\{ 1 - \frac{16}{(4+\lambda^2)^2} \right\} \cos \frac{2\pi x}{a} - \right. \\
 &\quad \left. - \frac{1}{\lambda^2} \left[\frac{\pi^2}{8} \frac{\left(\frac{\phi}{E} \right)}{j^2} \left\{ 1 - 2 \frac{\lambda^4}{(4+\lambda^2)^2} + \frac{\lambda^4}{(4+\lambda^2)^2} \right\} - \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \right] \left\{ \cos \pi \lambda - \frac{\sin(2\pi \lambda)}{\pi \lambda} \right\} \cos \frac{\pi x}{a} - \left(\frac{\sin(2\pi \lambda)}{\pi \lambda} \right) \left(\frac{\pi x}{a} \right) \sin \frac{\pi x}{a} \right\} \\
 &\quad \left. + \frac{1}{\lambda^2} \frac{\pi^2}{32} \frac{\left(\frac{\phi}{E} \right)}{j^2} \left\{ 1 - \frac{16\lambda^4}{(1+4\lambda^2)^2} \right\} \left\{ \left(\cos 2\pi \lambda - \frac{\sin(2\pi \lambda)}{2\pi \lambda} \right) \cos \frac{2\pi x}{a} - \left(\frac{\sin(2\pi \lambda)}{2\pi \lambda} \right) \left(\frac{2\pi x}{a} \right) \sin \frac{2\pi x}{a} \right\} \right\} \frac{1 + \frac{\sin(2\pi \lambda)}{2\pi \lambda}}{1 + \frac{\sin(4\pi \lambda)}{4\pi \lambda}}
 \end{aligned}$$

$$\begin{aligned}
 \left(\frac{\phi_y}{E} \right)_{y=b, x=0} &= -K + \frac{\lambda^2}{4} \left(\frac{\phi}{E} \right) \left\{ \frac{\pi^2}{8} \frac{\left(\frac{\phi}{E} \right)}{j^2} \left[\frac{5}{4} - \frac{2}{(1+\lambda^2)^2} + \frac{1}{(1+\lambda^2)^2} - \frac{4}{(4+\lambda^2)^2} \right] + \frac{1}{(1+\lambda^2)^2} \right. \\
 &\quad \left. - \frac{1}{\lambda^2} \left[\frac{\pi^2}{8} \frac{\left(\frac{\phi}{E} \right)}{j^2} \left\{ 1 - 2 \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{\lambda^4}{(4+\lambda^2)^2} \right\} - \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \right] \left(\cos \pi \lambda - \frac{\sin(2\pi \lambda)}{\pi \lambda} \right) \right. \\
 &\quad \left. + \frac{1}{\lambda^2} \frac{\pi^2}{32} \frac{\left(\frac{\phi}{E} \right)}{j^2} \left\{ 1 - \frac{16\lambda^4}{(1+4\lambda^2)^2} \right\} \left\{ \cos 2\pi \lambda - \frac{\sin(2\pi \lambda)}{2\pi \lambda} \right\} \right\} \frac{1 + \frac{\sin(2\pi \lambda)}{2\pi \lambda}}{1 + \frac{\sin(4\pi \lambda)}{4\pi \lambda}}
 \end{aligned}$$

Clamped, $y=0$ at $x=0$

$$F = E \left(\frac{a}{R} \right)^2 \frac{f}{g} \frac{1}{(a)^2} \left[\left(1 - \frac{a^2}{16} \right) \frac{1}{\lambda^2} \cos \frac{\pi x}{g} + \left(1 - \frac{a^2}{8} \right) \frac{\lambda^2}{(1+\lambda^2)^2} \cos \frac{\pi x}{a} \cos \frac{\pi x}{g} \right.$$

$$\left. - \frac{a^2}{16} \left\{ \lambda^2 \cos \frac{\pi x}{a} + \frac{a^2}{16} \cos \frac{2\pi x}{a} + \frac{1}{16\lambda^2} \cos \frac{2\pi x}{g} + \frac{\lambda^2}{(1+4\lambda^2)^2} \cos \frac{\pi x}{a} \cos \frac{2\pi x}{g} + \frac{\lambda^2}{(4+\lambda^2)^2} \cos \frac{2\pi x}{a} \cos \frac{\pi x}{g} \right\} \right.$$

$$\left. + a_0 \left(\frac{\pi x}{a} \right)^2 + b_0 \left(\frac{\pi x}{g} \right)^2 \right]$$

$$\phi_x = \frac{\partial F}{\partial x} = E \left(\frac{a}{R} \right)^2 \frac{f}{g} \left[\left(\frac{a^2}{16} - 1 \right) \cos \frac{\pi x}{g} \left(\frac{a^2}{g} - 1 \right) \frac{\lambda^4}{(1+\lambda^2)^2} \cos \frac{\pi x}{a} \cos \frac{\pi x}{g} \right.$$

$$\left. + \frac{a^2}{16} \left\{ \frac{1}{4} \cos \frac{2\pi x}{g} + \frac{4\lambda^4}{(1+4\lambda^2)^2} \cos \frac{\pi x}{a} \cos \frac{2\pi x}{g} + \frac{\lambda^4}{(4+\lambda^2)^2} \cos \frac{2\pi x}{a} \cos \frac{\pi x}{g} \right\} + 2b_0 \lambda^2 \right]$$

$$\frac{1}{2} \left[\left(\frac{\partial \phi}{\partial x} \right)^2 - \left(\frac{\partial \psi}{\partial x} \right)^2 \right] = \left(\frac{a}{R} \right)^2 \frac{f}{g} \left[- \left(\frac{\pi x}{a} \right) \sin \frac{\pi x}{a} \left(1 + \cos \frac{\pi x}{g} \right) + \frac{3a^2}{64} + \frac{a^2}{16} \cos \frac{\pi x}{g} + \frac{a^2}{64} \cos \frac{2\pi x}{g} \right.$$

$$\left. - \frac{3a^2}{64} \cos \frac{2\pi x}{a} - \frac{a^2}{16} \cos \frac{2\pi x}{a} \cos \frac{\pi x}{g} - \frac{a^2}{64} \cos \frac{2\pi x}{a} \cos \frac{2\pi x}{g} \right]$$

$$\begin{aligned} \frac{\partial U}{\partial \lambda} = & \left(\frac{a}{R} \right)^2 \frac{f}{g} \left[+ \left(\frac{\pi \lambda}{a} \right) \sin \frac{\pi x}{a} - \frac{3}{64} (4\pi^2) + \frac{3}{64} (4\pi^2) \cos \frac{2\pi x}{a} + 2b_0 \lambda^2 \right. \\ & + \left\{ \left(\frac{\pi \lambda}{a} \right) \sin \frac{\pi x}{a} - 1 + \left(\frac{f\pi^2}{g} - 1 \right) \frac{\lambda^4}{(1+\lambda^2)^2} \cos \frac{\pi x}{a} + \frac{f\pi^2}{16} \left(\frac{\lambda^4}{(4+\lambda^2)^2} + 1 \right) \cos \frac{2\pi x}{a} \right\} \cos \frac{\pi x}{g} + \\ & \left. + \frac{f\pi^2}{16} \left\{ \frac{4\lambda^4}{(1+4\lambda^2)^2} \cos \frac{\pi x}{a} + \frac{f}{4} \cos \frac{2\pi x}{a} \right\} \cos \frac{2\pi x}{g} \right] \end{aligned}$$

$$U=0 \text{ at } x=a, \text{ gives } 1 - \frac{3}{64} f\pi^2 + 2b_0 \lambda^2 = 0$$

$$\boxed{2b_0 \lambda^2 = \left(\frac{3}{64} f\pi^2 - 1 \right)}$$

$$\begin{aligned} \tilde{y} = \frac{\partial F}{\partial \lambda^2} = & E \left(\frac{a}{R} \right)^2 \frac{f}{g} \left[\left(\frac{f\pi^2}{g} - 1 \right) \frac{\lambda^2}{(1+\lambda^2)^2} \cos \frac{\pi x}{a} \cos \frac{\pi x}{g} + \frac{f\pi^2}{16} \left\{ \lambda^2 \cos \frac{\pi x}{a} + \frac{\lambda^2}{4} \cos \frac{2\pi x}{a} \right. \right. \\ & \left. \left. + \frac{\lambda^2}{(1+4\lambda^2)^2} \cos \frac{\pi x}{a} \cos \frac{2\pi x}{g} + \frac{4\lambda^2}{(4+\lambda^2)^2} \cos \frac{2\pi x}{a} \cos \frac{\pi x}{g} \right\} + 2a_0 \right] \end{aligned}$$

$$\boxed{-\frac{\sigma}{E} = \left(\frac{a}{R} \right)^2 \frac{f}{g} (2a_0)}$$

$$\left(\frac{V}{b}\right)_{y=b} = -\left(\frac{a}{R}\right)^2 \frac{\lambda^2}{8} \frac{3}{64} (\frac{\pi}{2})^2 - \frac{\sigma}{E}$$

$$\boxed{\frac{\Delta b}{E a b L} = -\left(\frac{\sigma}{E}\right) - \left(\frac{a}{R}\right)^2 \left(\frac{\pi}{8}\right)^2 \left(\frac{3}{8} \pi \lambda^2\right) \frac{\sigma}{E}}$$

$$\frac{\sigma_x}{E} = \left(\frac{a}{R}\right)^2 \frac{\pi}{8} \left[\left(\frac{3}{64} \pi^2 - 1\right) + \left\{ \left(\frac{-\pi^2}{16} - 1\right) + \left(\frac{\pi^2}{8} - 1\right) \frac{\lambda^4}{(1+\lambda^2)^2} \cos \frac{\pi x}{a} + \frac{\pi^2}{16} \frac{\lambda^4}{(4+\lambda^2)^2} \cos \frac{2\pi x}{a} \right\} \cos \frac{\pi y}{b} \right. \\ \left. + \frac{\pi^2}{16} \left[\frac{1}{4} + \frac{4\lambda^4}{(1+4\lambda^2)^2} \cos \frac{\pi x}{a} \right] \cos \frac{2\pi y}{b} \right]$$

$$\frac{1}{2ab} \int_0^a \int_0^b \left(\frac{\sigma_x}{E}\right) dx dy = \left(\frac{a}{R}\right)^2 \left(\frac{\pi}{8}\right)^2 \left[\frac{1}{2} \left(\frac{3}{64} \pi^2 - 1\right)^2 + \frac{1}{4} \left(\frac{-\pi^2}{16} - 1\right)^2 + \frac{1}{8} \left(\frac{-\pi^2}{8} - 1\right)^2 \frac{\lambda^8}{(1+\lambda^2)^4} \right. \\ \left. + \frac{1}{8} \left(\frac{\pi^2}{16}\right)^2 \frac{\lambda^8}{(4+\lambda^2)^4} + \left(\frac{-\pi^2}{16}\right)^2 \left\{ \frac{1}{4} \frac{1}{16} + \frac{1}{8} \frac{16\lambda^8}{(1+4\lambda^2)^4} \right\} \right]$$

$$\frac{\sigma_y}{E} = -\frac{\sigma}{E} + \left(\frac{a}{R}\right)^2 \frac{\pi}{8} \left[\frac{-\pi^2}{16} \lambda^2 \cos \frac{\pi x}{a} + \frac{\pi^2}{64} \lambda^2 \cos \frac{2\pi x}{a} \right. \\ \left. + \left\{ \left(\frac{-\pi^2}{8} - 1\right) \frac{\lambda^2}{(1+\lambda^2)^2} \cos \frac{\pi x}{a} + \frac{\pi^2}{16} \frac{4\lambda^2}{(4+\lambda^2)^2} \cos \frac{2\pi x}{a} \right\} \cos \frac{\pi y}{b} \right. \\ \left. + \frac{\pi^2}{16} \frac{\lambda^2}{(1+\lambda^2)^2} \cos \frac{\pi x}{a} \cos \frac{2\pi y}{b} \right]$$

$$\frac{1}{2ab} \int_0^a \int_0^b \left(\frac{Q}{E} \right)^2 dx dy = \frac{1}{2} \left(\frac{Q}{E} \right)^2 + \left(\frac{Q}{R} \right)^4 \left[\frac{1}{4} \left(\frac{a^2}{16} \right)^2 + \frac{1}{4} \left(\frac{a^2}{64} \right)^2 \right] \\ + \frac{1}{8} \left(\frac{a^2}{8} - 1 \right)^2 \frac{b^4}{(1+b^2)^4} + \frac{1}{8} \left(\frac{a^2}{16} \right) \frac{b^4}{(4+b^2)^4} + \frac{1}{8} \left(\frac{a^2}{16} \right)^2 \frac{b^4}{(1+4b^2)^4} \Big]$$

$$\frac{U_H}{E} = \left(\frac{Q}{R} \right)^2 \frac{1}{8} \left[\left(\frac{a^2}{8} - 1 \right) \frac{b^3}{(1+b^2)^2} + \frac{a^3}{16} \frac{b^3}{(1+4b^2)^2} + \frac{a^3}{16} \frac{b^3}{(1+4b^2)^2} \cos \frac{\pi x}{a} \cos \frac{2\pi y}{b} + \frac{a^2}{16} \frac{b^2}{(4+b^2)^2} \cos \frac{2\pi x}{a} \cos \frac{2\pi y}{b} \right]$$

$$= \left(\frac{Q}{R} \right)^2 \frac{1}{8} \left[\left\{ \left(\frac{a^2}{8} - 1 \right) \frac{b^3}{(1+b^2)^2} + \frac{a^3}{16} \frac{b^3}{(1+4b^2)^2} \cos \frac{\pi x}{a} + \frac{a^2}{16} \frac{b^2}{(4+b^2)^2} \cos \frac{2\pi x}{a} \right\} \cos \frac{\pi y}{b} \right. \\ \left. + \frac{a^2}{16} \frac{b^2}{(1+4b^2)^2} \cos \frac{\pi x}{a} \cos \frac{2\pi y}{b} \right]$$

$$\frac{1}{ab} \int_0^a \int_0^b \left(\frac{U_H}{E} \right)^2 dx dy = \left(\frac{Q}{R} \right)^4 \left(\frac{1}{8} \right)^2 \left[\frac{1}{4} \left(\frac{a^2}{8} - 1 \right)^2 \frac{b^6}{(1+b^2)^2} + \frac{1}{4} \left(\frac{a^2}{16} \right)^2 \frac{b^6}{(4+b^2)^2} + \frac{1}{4} \left(\frac{a^2}{16} \right)^2 \frac{b^6}{(1+4b^2)^2} \right]$$

$$\frac{1}{2} \left(\frac{3}{64} \pi^2 - 1 \right)^2 + \frac{1}{4} \left(\frac{1}{16} \pi^2 - 1 \right)^2 + \frac{1}{4} \left(\frac{1}{16} \pi^2 \right)^2 + \frac{1}{4} \left(\frac{1}{64} \pi^2 \right)^2 + \frac{1}{8} \left(\frac{1}{8} \pi^2 - 1 \right)^2 \frac{\lambda^4}{(1+\lambda^2)^2} \\ + \frac{1}{8} \left(\frac{1}{16} \pi^2 \right)^2 \frac{\lambda^4}{(4+\lambda^2)^2} + \left(\frac{1}{16} \pi^2 \right)^2 \frac{1}{64} + \frac{1}{8} \left(\frac{1}{16} \pi^2 \right)^2 \frac{\lambda^4}{(1+\lambda^2)^2}$$

$$H_1(\lambda) = \frac{1}{2} \left(\frac{3}{64} \right)^2 + \frac{1}{4} \left(\frac{1}{16} \right)^2 + \frac{1}{4} \left(\frac{1}{16} \right)^2 + \frac{1}{4} \left(\frac{1}{64} \right)^2 + \frac{1}{8} \frac{1}{64} \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{1}{8} \frac{1}{256} \frac{\lambda^4}{(4+\lambda^2)^2} \\ + \frac{1}{8} \frac{1}{256} \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{1}{64} \frac{1}{256}$$

$$H_1'(\lambda) = \frac{35}{16384} + \frac{17}{16384} \lambda^4 + \frac{1}{512} \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{1}{2048} \frac{\lambda^4}{(4+\lambda^2)^2} + \frac{1}{2048} \frac{\lambda^4}{(1+\lambda^2)^2}$$

$$H_2(\lambda) = \frac{3}{64} + \frac{1}{32} + \frac{1}{32} \frac{\lambda^4}{(1+\lambda^2)^2}$$

$$H_2'(\lambda) = \frac{5}{64} + \frac{1}{32} \frac{\lambda^4}{(1+\lambda^2)^2}$$

$$H_3(\lambda) = \frac{3}{4} + \frac{1}{8} \frac{\lambda^4}{(1+\lambda^2)^2}$$

$$\begin{aligned} \text{The binding energy} &= \frac{1}{24} \left(\frac{f}{R}\right)^2 \left(\frac{f}{\delta}\right)^2 \pi^4 \left\{ \frac{3}{4} (1+\lambda^4) + \frac{1}{2} \lambda^2 \right\} \\ &= \left(\frac{f}{R}\right)^2 \left(\frac{f}{\delta}\right)^2 \pi^4 \left\{ \frac{1}{32} (1+\lambda^4) + \frac{\lambda^2}{48} \right\} \end{aligned}$$

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The total potential of the system

$$\begin{aligned} &= \left(\frac{A}{R}\right)^4 \left(\frac{f}{\delta}\right)^2 \left[H_1 (A\pi^2)^2 - H_2 (A\pi^2) + H_3 \right] + \left(\frac{f}{R}\right)^2 \left(\frac{f}{\delta}\right)^2 \pi^4 \left\{ \frac{1+\lambda^4}{32} + \frac{\lambda^2}{48} \right\} \\ &\quad - \frac{1}{2} \left(\frac{\sigma}{E}\right)^2 - \left(\frac{A}{R}\right)^2 \left(\frac{f}{\delta}\right)^2 \frac{\sigma}{E} \left(\frac{3}{\delta} \pi^2 \lambda^2\right) \end{aligned}$$

Thus

$$\begin{aligned} \frac{3}{\delta} \frac{\sigma}{E} \pi^2 \lambda^2 &= \left(\frac{A}{R}\right)^2 \left[2H_1 (A\pi^2)^2 - \frac{3}{2} H_2 (A\pi^2) + H_3 \right] \\ &\quad + \left(\frac{f}{R}\right)^2 \pi^4 \left\{ \frac{1+\lambda^4}{32} + \frac{\lambda^2}{48} \right\} \frac{1}{\left(\frac{\sigma}{R}\right)^2} \end{aligned}$$

$$\begin{aligned} \lambda^2 K &= \gamma^2 \left[\frac{16}{3} H_1 f^2 \pi^2 - 4 H_2 f + \frac{f}{3} H_3 \frac{1}{\pi^2} \right] + \frac{\pi^2}{\gamma^2} \left[\frac{1+\lambda^4}{12} + \frac{\lambda^2}{18} \right] \\ &= \frac{\pi^2}{f^2} \left[\frac{64}{3} H_1 \left(\frac{f}{E}\right)^2 + \left(\frac{1+\lambda^4}{12} + \frac{\lambda^2}{18}\right) \right] + \frac{f^2}{\pi^2} \frac{f}{3} H_3 - f H_2 \left(\frac{f}{E}\right) \end{aligned}$$

$$\lambda^2 K = 2 \left[\frac{512}{9} H_1 H_3 \left(\frac{f}{E}\right)^2 + H_3 \left\{ \frac{2}{9} (1+\lambda^4) + \frac{4\lambda^2}{27} \right\} \right]^{\frac{1}{2}} - f H_2 \left(\frac{f}{E}\right)$$

$$\left[f^2 = \pi^2 \left[\frac{f H_1}{H_3} \left(\frac{f}{E}\right)^2 + \frac{1}{H_3} \left\{ \frac{1}{32} (1+\lambda^4) + \frac{\lambda^2}{48} \right\} \right] \right]$$

thus

$$\varepsilon = - \left(\frac{A}{R}\right)^2 \frac{1}{f} \frac{\lambda^2}{64} (f\pi^2) - \frac{5}{E}$$

$$\frac{\varepsilon}{\left(\frac{A}{R}\right)^2} = -K - \frac{3}{128} \pi^2 \lambda^2 \frac{\left(\frac{f}{E}\right)^2}{f^2}$$

$$\left(\frac{\varepsilon}{\left(\frac{A}{R}\right)^2}\right) = -K - \frac{3}{128} \lambda^2 \left(\frac{f}{E}\right)^2 \frac{1}{\sqrt{\frac{8H_1}{H_3} \left(\frac{f}{E}\right)^2 + \frac{1}{H_3} \left(\frac{1+\lambda^2}{32} + \frac{1^2}{48}\right)}}$$

$$K = 2 \left\{ A \left(\frac{f}{E}\right)^2 + B \right\}^{\frac{1}{2}} - C \left(\frac{f}{E}\right)$$

$$2A \left(\frac{f}{E}\right) = C \left\{ A \left(\frac{f}{E}\right)^2 + B \right\}^{\frac{1}{2}}$$

$$(4A^2 - C^2 A) \left(\frac{f}{E}\right)^2 = C^2 B \quad \left(\frac{f}{E}\right)^2 = \frac{C^2 B}{4A^2 - C^2 A} = \frac{B}{\left(\frac{2A}{C}\right)^2 - A}$$

$$H_1' = \frac{1}{2048} \left[\frac{35}{8} + \frac{17}{8} \lambda^4 + 4 \frac{\lambda^4}{(1+\lambda^2)^2} + \frac{\lambda^4}{(4+\lambda^2)^2} + \frac{\lambda^4}{(1+4\lambda^2)^2} \right], \quad H_2' = \frac{1}{32} \left[2.5 + \frac{\lambda^4}{(1+\lambda^2)^2} \right]$$

λ	λ^4	$\frac{\lambda^4}{(1+\lambda^2)^2}$	$\frac{\lambda^4}{(4+\lambda^2)^2}$	$\frac{\lambda^4}{(1+4\lambda^2)^2}$	2048 H_1	$\frac{\lambda^4}{(1+4\lambda^2)^2}$	H_1'/λ^2	32 H_2	H_2'/λ^2
0.05	0.0000625	0.00000622	0.00000039	0.00000039	4.375044	0.00000613	0.654501	2.500006	31.2500
0.10	0.0001	0.00009803	0.0000622	0.0000622	4.375703	0.00009246	0.213557	2.500098	7.812806
0.15	0.00050625	0.00048422	0.0003129	0.0003129	4.38440	0.0004610	0.0950189	2.500484	3.472194
0.20	0.0016	0.00147929	0.0009803	0.0009803	4.385604	0.00118906	0.0535352	2.501479	1.95440
0.30	0.0081	0.00681761	0.00048422	0.00048422	4.424346	0.00437933	0.0240036	2.506818	0.8766229
0.40	0.0256	0.01902497	0.00147929	0.00147929	4.516497	0.00951814	0.0137833	2.519025	0.491991
0.50	0.0625	0.040000	0.00346021	0.00346021	4.686898	0.01512500	0.00915410	2.54000	0.3125000
0.60	0.1296	0.07006920	0.00681761	0.00681761	4.959263	0.02176834	0.00672643	2.570069	0.2230963
0.80	0.4096	0.15229030	0.01902497	0.01902497	5.905905	0.03231915	0.0045085	2.652210	0.1295064
1.00	1.0000	0.250000	0.040000	0.040000	7.580000	0.040000	0.00370117	2.750000	0.0859375
1.20	2.0736	0.34821347	0.07006920	0.07006920	10.290020	0.04537356	0.00348918	2.848213	0.0618119
1.40	3.8416	0.43845873	0.10141428	0.10141428	14.448543	0.04915152	0.00359971	2.938459	0.0468504
1.60	6.5536	0.51710643	0.15229030	0.15229030	20.537990	0.05187371	0.00392418	3.017106	0.0368299
1.80	10.4976	0.58392667	0.20026861	0.20026861	29.272242	0.05386655	0.00441445	3.063927	0.0292447
2.00	16.000	0.640000	0.250000	0.250000	41.240363	0.05536332	0.00503422	3.140000	0.0245313
2.20	23.4256	0.68214452	0.29971814	0.29971814	57.237978	0.05651124	0.00574457	3.182135	0.0205461

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$$H_3 = \frac{1}{8} \left(6 + \frac{\lambda^4}{(1+\lambda^2)^2} \right)$$

λ	H_3/λ^2	$\frac{\frac{1}{8}(1+\lambda^4) + \frac{6}{32}\lambda^4}{\lambda^2}$	A	B	C	K_0	$\left(\frac{2A}{C}\right)^2 - A$	$(\delta/\epsilon)_{min}^2$
0.05	300.000	88.0345	14583.5	24711.25	250.0000		-942.31	
0.10	75.0001	223726	911603	167795	625024		-60702	
0.15	333860	10.0297	180.198	334350	277832		-11.933	
0.20	187566	571259	57.1182	107137	156342		-37284	
0.30	8.34280	263728	11.3924	22.0023	6.96338		-0.68583	
0.40	4.70236	157259	3.66720	7.39488	3.93598		-0.17666	
0.50	3.02000	1.09259	1.57272	3.29162	2.54000		-0.03918	
0.60	2.10766	0.845432	0.806515	1.78188	1.78477		+0.010294	173.099
0.80	1.201619	0.637593	0.308014	0.716144	1.03605		+0.04557	168.283
1.00	0.781250	0.592593	0.164496	0.462163	0.682500	1.3608	+0.064498	77.7794
1.20	0.551067	0.622469	0.109384	0.343022	0.494495	1.1716	+0.061340	39.7292
1.40	0.410616	0.697082	0.0840774	0.286233	0.374803	1.0696	+0.112446	24.4130
1.60	0.318218	0.703843	0.0710397	0.255797	0.294639	1.0116	+0.161491	15.8397
1.80	0.254010	0.936735	0.0637470	0.237960	0.237158	0.9760	+0.223317	10.6548
2.00	0.203500	1.092593	0.0594262	0.221713	0.19250	0.9522	+0.307346	07.37647
2.20	0.172592	1.261617	0.0566760	0.219126	0.164369	0.9360	+0.419246	05.22667

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λ	$(\delta t)_{min}$	K_{min}	λ	$(\frac{\delta}{t})^*$	K	$\frac{8H_1}{H_3} = D$	E	$D(\frac{\delta}{t})^2 + E$
0.05								
0.10								
0.15								
0.20								
0.30			0.3	4.82	0.303 0.0645	0.023017	0.04445	0.5792
0.40			0.4	4.53	0.400 0.0625	0.023449	0.04403	0.5282
0.50			0.5	4.13	0.486 0.146	0.024249	0.05088	0.4645
0.60	13.567	0.2997	0.6	3.48	0.583 0.165	0.025531	0.05641	0.3656
0.80	4.1022	0.6282	0.8	2.55	0.690 0.1845	0.030000	0.07461	0.2697
1.00	2.6792	0.7222	1.0	1.88	0.756 0.165	0.039900	0.10667	0.2407
1.20	1.9932	0.7780	1.2	1.30	0.816 0.152	0.050653	0.15885	0.2445
1.40	1.5625	0.8166	1.4	1.00	0.841 0.082	0.070133	0.23873	0.3088
1.60	1.2585	0.8429	1.6					
1.80	1.0322	0.8605	1.8					
2.00	0.8586	0.8717	2.0					
2.20	0.7226	0.8787	2.20		0.925			

(E) Shortening due to buckling !!

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$$\lambda = 2.20$$

$$\text{at } \left(\frac{f}{E}\right) = 0.5,$$

$$K = 2 \left\{ 0.0566980 \times 0.25 + 0.219126 \right\}^{\frac{1}{2}} - 0.5 \times 0.164369$$

$$= 0.8838$$

$$\left(\frac{f}{E}\right) = 1.0$$

$$K = 0.8860$$

$$\lambda = 2.00 \text{ at } \left(\frac{f}{E}\right) = 0.5$$

$$K = 2 \sqrt{0.241570} - 0.5 \times 0.196250 = 0.8849$$

$$\left(\frac{f}{E}\right) = 1.0$$

$$K = 2 \sqrt{0.286139} - 0.196250 = 0.8736$$

$$\lambda = 1.8 \quad \left(\frac{f}{E}\right) = 0.5$$

$$K = 2 \sqrt{0.253877} - 0.5 \times 0.237958 = 0.8887$$

$$\left(\frac{f}{E}\right) = 2.0$$

$$K = 2 \sqrt{0.492946} - 2 \times 0.237958 = 0.9283$$

$$\lambda = 1.6 \quad \left(\frac{f}{E}\right) = 1.0$$

$$K = 2 \sqrt{0.326837} - 0.294639 = 0.8876$$

$$\left(\frac{f}{E}\right) = 2.0$$

$$K = 2 \sqrt{0.539956} - 2 \times 0.294639 = 0.8804$$

$$\lambda = 1.4, \quad \left(\frac{f}{E}\right) = 1.0$$

$$K = 2 \sqrt{0.370320} - 0.374803 = 0.8423$$

$$\left(\frac{f}{E}\right) = 2.0$$

$$K = 2 \sqrt{0.622583} - 2 \times 0.374803 = 0.8285$$

$$\lambda = 1.2, \quad \left(\frac{f}{E}\right) = 1.0$$

$$K = 2 \sqrt{0.452406} - 0.494495 = 0.8507$$

$$\left(\frac{f}{E}\right) = 3.0$$

$$K = 2 \sqrt{1.327478} - 3 \times 0.494495 = 0.8208$$

$$\lambda = 1.0 \quad \left(\frac{d}{E}\right) = 1.00 \quad K = 2\sqrt{0.627459} - 0.647500 = 0.1967 \quad \underline{\underline{570}}$$

$$\left(\frac{d}{E}\right) = 3.00 \quad K = 2\sqrt{1.943427} - 3 \times 0.647500 = 0.7256$$

$$\lambda = 0.8 \quad \left(\frac{d}{E}\right) = 1.00 \quad K = 2\sqrt{1.074158} - 1.03605 = 1.0368$$

$$\left(\frac{d}{E}\right) = 3.00 \quad K = 2\sqrt{3.538270} - 3 \times 1.03605 = 0.6539$$

$$\left(\frac{d}{E}\right) = 5.00 \quad K = 2\sqrt{8.466494} - 5 \times 1.03605 = 0.6392$$

$$\lambda = 0.6 \quad \left(\frac{d}{E}\right) = 3.00 \quad K = 2\sqrt{9.04052} - 3 \times 1.78477 = 0.6592$$

$$\left(\frac{d}{E}\right) = 5.00 \quad K = 2\sqrt{21.94476} - 5 \times 1.78477 = 0.4452$$

$$\left(\frac{d}{E}\right) = 7.00 \quad K = 2\sqrt{41.30112} - 7 \times 1.78477 = 0.3598$$

$$\left(\frac{d}{E}\right) = 10.00 \quad K = 2\sqrt{82.43338} - 10 \times 1.78477 = 0.3109$$

$$\left(\frac{d}{E}\right) = 15.00 \quad K = 2\sqrt{183.24776} - 15 \times 1.78477 = 0.3023$$

$$\lambda = 0.5 \quad \left(\frac{d}{E}\right) = 3.00 \quad K = 2\sqrt{17.45410} - 3 \times 2.54 = 0.7356$$

$$\left(\frac{d}{E}\right) = 4.00 \quad K = 2\sqrt{28.46314} - 4 \times 2.54 = 0.5102$$

$$\left(\frac{d}{E}\right) = 5.00 \quad K = 2\sqrt{42.61762} - 5 \times 2.54 = 0.3564$$

$$\lambda=0, \quad H_1 = \frac{35}{16384}, \quad H_2 = \frac{5}{64}, \quad H_3 = \frac{3}{4} \quad \underline{571}$$

$$\lambda^2 K = 2 \left[\frac{35 \times \left(\frac{d}{L}\right)^2}{32 \times 3 \times 4} + \frac{3}{4} \times \frac{2}{9} \right]^{\frac{1}{2}} - \frac{5}{8} \left(\frac{d}{L}\right)$$

$$\frac{35}{32 \times 12} \left(\frac{d}{L}\right)^2 + \frac{1}{6} = \frac{25}{256} \left(\frac{d}{L}\right)^2$$

$$\left(\frac{d}{L}\right)^2 \left[\frac{25}{256} - \frac{35}{32 \times 12} \right] = \frac{1}{6}, \quad \left(\frac{d}{L}\right)^2 = \frac{128}{5} = 25.6$$

$$\left(\frac{d}{L}\right) = 5.0597$$

$$\lambda = 0.4, \quad \left(\frac{d}{L}\right) = 4.00 \quad K = 2\sqrt{66.39008} - 4 \times 3.93598 = 0.5521$$

$$\left(\frac{d}{L}\right) = 5.00 \quad K = 2\sqrt{99.57418} - 5 \times 3.93598 = 0.2775$$

$$\left(\frac{d}{L}\right) = 6.00 \quad K = 2\sqrt{140.13408} - 6 \times 3.93598 = 0.0598$$

$$\lambda = 0.3 \quad \left(\frac{d}{L}\right) = 4.00 \quad K = 2\sqrt{204.2807} - 4 \times 6.96338 = 0.7319$$

$$\left(\frac{d}{L}\right) = 5.00 \quad K = 2\sqrt{306.8123} - 5 \times 6.96338 = 0.2152$$

$$\left(\frac{d}{L}\right) = 6.00 \quad K = 2\sqrt{432.1287} - 6 \times 6.96338 = -0.2049$$

Clamped, zero displacement at edges

$$K = \frac{\pi^2}{f^2} \left[\frac{64}{3} \left(\frac{f}{E} \right)^2 \left\{ \frac{35}{16384} \lambda^2 + \frac{17}{16384} \lambda^2 + \frac{1}{512} \frac{\lambda^2}{(1+\lambda^2)^2} + \frac{1}{2048} \frac{\lambda^2}{(4+\lambda^2)^2} + \frac{1}{7048} \frac{\lambda^2}{(1+4\lambda^2)^2} \right\} \right.$$

$$+ \left\{ \frac{1}{12} \lambda^2 + \frac{1}{12} \lambda^2 + \frac{1}{18} \right\} + \frac{f^2}{\pi^2} \left\{ \frac{1}{4} \lambda^2 + \frac{1}{8} \frac{\lambda^2}{(1+\lambda^2)^2} \right\} - 8 \left(\frac{f}{E} \right) \left\{ \frac{1}{64} \lambda^2 + \frac{1}{32} \frac{\lambda^2}{(1+\lambda^2)^2} \right\}$$

$$= \frac{\pi^2}{f^2} \left[W^2 \left\{ \frac{35}{768} \lambda^2 + \frac{17}{768} \lambda^2 + \frac{1}{24} \frac{\lambda^2}{(1+\lambda^2)^2} + \frac{1}{96} \frac{\lambda^2}{(4+\lambda^2)^2} + \frac{1}{96} \frac{\lambda^2}{(1+4\lambda^2)^2} \right\} \right.$$

$$+ \frac{1}{6} \left(\frac{1}{2} \lambda^2 + \frac{\lambda^2}{2} + \frac{1}{3} \right) + \frac{f^2}{\pi^2} \left\{ \frac{1}{\lambda^2} + \frac{1}{3} \frac{\lambda^2}{(1+\lambda^2)^2} \right\} - W \left\{ \frac{5}{8} \lambda^2 + \frac{1}{4} \frac{\lambda^2}{(1+\lambda^2)^2} \right\}$$

$$\frac{\partial K}{\partial \lambda^2} = 0 ; 0 = \frac{\pi^2}{f^2} \left[W^2 \left\{ -\frac{35}{768} \lambda^4 + \frac{17}{768} \lambda^4 + \frac{1}{24} \frac{1}{(1+\lambda^2)^2} - \frac{1}{48} \frac{\lambda^2}{(1+\lambda^2)^3} + \frac{1}{96} \frac{1}{(4+\lambda^2)^2} - \frac{1}{192} \frac{\lambda^2}{(4+\lambda^2)^3} \right. \right.$$

$$+ \frac{1}{96} \frac{1}{(1+4\lambda^2)^2} - \frac{1}{192} \frac{4\lambda^2}{(1+4\lambda^2)^3} \left. \right\} + \frac{1}{6} \left(-\frac{1}{2} \lambda^4 + \frac{1}{2} \right) + \frac{f^2}{\pi^2} \left\{ -\frac{2}{\lambda^4} + \frac{1}{3} \frac{1}{(1+\lambda^2)^2} - \frac{1}{6} \frac{\lambda^2}{(1+\lambda^2)^3} \right\} - W \left\{ -\frac{5}{8} \lambda^4 + \frac{1}{4} \frac{1}{(1+\lambda^2)^2} - \frac{1}{8} \frac{\lambda^2}{(1+\lambda^2)^3} \right\}$$

$$0 = \frac{\pi^2}{f^2} \left[W \left\{ -\frac{35}{768} \frac{1}{\lambda^4} + \frac{17}{48} + \frac{1}{48} \frac{2+\lambda^2}{(1+\lambda^2)^3} + \frac{1}{192} \frac{8+\lambda^2}{(4+\lambda^2)^3} + \frac{1}{192} \frac{2+4\lambda^2}{(1+4\lambda^2)^3} \right\} \right.$$

$$\left. + \frac{1}{12} \left(1 - \frac{1}{\lambda^4} \right) \right] + \frac{f^2}{\pi^2} \left\{ -\frac{2}{\lambda^4} + \frac{1}{6} \frac{2+\lambda^2}{(1+\lambda^2)^3} \right\} - W \left\{ -\frac{5}{8} \frac{1}{\lambda^4} + \frac{1}{8} \frac{2+\lambda^2}{(1+\lambda^2)^3} \right\}$$

$$0 = \frac{\pi^2}{f^2} \left(\frac{17}{768} W + \frac{1}{12} \right) - \left[\frac{\pi^2}{f^2} \left(\frac{35}{768} W + \frac{1}{12} \right) + 2 \frac{f^2}{\pi^2} - \frac{5}{8} W \right] \frac{1}{\lambda^4}$$

$$+ \left[\frac{\pi^2}{f^2} \frac{1}{48} W + \frac{1}{6} \frac{f^2}{\pi^2} - \frac{1}{8} W \right] \frac{2+\lambda^2}{(1+\lambda^2)^3} + \left(\frac{\pi^2}{f^2} \frac{1}{192} W \right) \frac{8+\lambda^2}{(4+\lambda^2)^3} + \left(\frac{\pi^2}{f^2} \frac{1}{192} W \right) \frac{2+4\lambda^2}{(1+4\lambda^2)^3}$$

$$0 = \frac{\pi^2}{f^2} \left(\frac{17}{768} W + \frac{1}{12} \right) \lambda^4 (1+\lambda^2)^3 (4+\lambda^2)^3 - \left[\frac{\pi^2}{f^2} \left(\frac{35}{768} W + \frac{1}{12} \right) + 2 \frac{f^2}{\pi^2} - \frac{5}{8} W \right] (1+\lambda^2)^3 (4+\lambda^2)^3 (1+4\lambda^2)^3$$

$$+ \left[\frac{\pi^2}{f^2} \frac{1}{48} W + \frac{1}{6} \frac{f^2}{\pi^2} - \frac{1}{8} W \right] \lambda^4 (2+\lambda^2) (1+4\lambda^2)^3 (4+\lambda^2)^3 + \left(\frac{\pi^2}{f^2} \frac{1}{192} W \right) \lambda^4 (8+\lambda^2) (1+\lambda^2)^3 (1+4\lambda^2)^3$$

$$+ \left(\frac{\pi^2}{f^2} \frac{1}{192} W \right) \lambda^4 (2+4\lambda^2) (1+\lambda^2)^3 (1+4\lambda^2)^3$$

A 11th order equation for λ^2 ...

for any given value of f^2 the

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$$K = \frac{\pi^2}{f^2} \left[\frac{64}{3} \frac{H_1}{\lambda^2} \left(\frac{f}{E} \right)^2 + \frac{3}{8} \frac{1}{\lambda^2} \left(\frac{2}{9} (1 + \lambda^2) + \frac{4}{27} \lambda^2 \right) \right] + \frac{f^2}{\pi^2} \frac{8}{3} \frac{H_2}{\lambda^2} - \frac{8H_2}{\lambda^2} \left(\frac{f}{E} \right)$$

$$= \frac{\pi^2}{f^2} \left[A(\lambda^2) \left(\frac{f}{E} \right)^2 + B(\lambda^2) \right] + \frac{f^2}{\pi^2} C(\lambda^2) - D(\lambda^2) \left(\frac{f}{E} \right)$$

λ	A	B	C	D	B+C		
0.05	18.2294	33.3891	800.000	250.000	833.389		
0.10	4.55802	8.38973	200.000	62.5000	208.390		
0.15	2.02707	3.76114	88.8960	27.4315	92.6571		
0.20	1.14208	2.14222	50.0123	15.6342	52.1545		
0.30	0.512087	0.988980	22.2475	6.96338	23.2365		
0.40	0.294044	0.589721	12.5396	3.93598	13.1293		
0.50	0.195287	0.409721	8.05333	2.54000	8.46305		
0.60	0.143497	0.317037	5.62043	1.78477	5.93747		
0.80	0.096125	0.239097	3.20432	1.03605	3.44342		
1.00	0.078958	0.222222	2.08333	0.68750	2.30555		
1.20	0.074436	0.233426	1.46951	0.494495	1.70294		
1.40	0.076794	0.261406	1.09498	0.374403	1.35639		
1.60	0.083716	0.301441	0.848581	0.294639	1.15002		
1.80	0.094111	0.351275	0.677360	0.237958	1.02464		
2.00	0.107397	0.409722	0.553333	0.196250	0.96305		
2.20	0.123191	0.476106	0.460245	0.164369	0.93635		

Take $\frac{\pi^2}{f^2} = 1$

$$K = A \left(\frac{f}{E} \right)^2 - D \left(\frac{f}{E} \right) + (B+C)$$

We have

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$$\sigma_x = \frac{E}{1-\nu^2} \left\{ \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 + \nu \left[\frac{\partial v}{\partial y} - \frac{w}{R} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \right] \right\} \quad (1)$$

where $y = R\theta$

$$\sigma_y = \frac{E}{1-\nu^2} \left\{ \frac{\partial v}{\partial y} - \frac{w}{R} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 + \nu \left[\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right] \right\} \quad (2)$$

$$\tau_{xy} = \frac{E}{2(1+\nu)} \left\{ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right\} \quad (3)$$

$$(1) - (2) \cdot \nu,$$

$$\sigma_x - \nu \sigma_y = \frac{E}{1-\nu^2} \left\{ (1-\nu^2) \left[\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right] \right\}$$

$$\sigma_y - \nu \sigma_x = \frac{E}{1-\nu^2} (1-\nu^2) \left[\frac{\partial v}{\partial y} - \frac{w}{R} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \right]$$

Therefore $\frac{\partial u}{\partial x} = \frac{1}{E} (\sigma_x - \nu \sigma_y) - \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \quad (4)$

$$\frac{\partial v}{\partial y} = \frac{1}{E} (\sigma_y - \nu \sigma_x) - \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 + \frac{w}{R} \quad (5)$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{2(1+\nu)}{E} \tau_{xy} - \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \quad (6)$$

Due to the equilibrium condition

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y}$$

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From (4), (5), (6)

$$\begin{aligned} & \frac{1}{E} \left(\frac{\partial^2 \sigma_x}{\partial y^2} - \nu \frac{\partial^2 \sigma_y}{\partial y^2} + \frac{\partial^2 \sigma_y}{\partial x^2} - \nu \frac{\partial^2 \sigma_x}{\partial x^2} \right) - \left(\frac{\partial^2 \tau_{xy}}{\partial x \partial y} \right)^2 - \frac{\partial \omega}{\partial x} \frac{\partial^2 \omega}{\partial x \partial y^2} \\ & \quad - \left(\frac{\partial^2 \omega}{\partial x \partial y} \right)^2 - \frac{\partial \omega}{\partial y} \frac{\partial^3 \omega}{\partial x^2 \partial y} + \frac{1}{R} \frac{\partial^2 \omega}{\partial x^2} \\ &= \frac{2(1+\nu)}{E} \frac{\partial^2 \tau_{xy}}{\partial x \partial y} - \frac{2}{\partial y} \left(\frac{\partial \omega}{\partial x} \frac{\partial^2 \omega}{\partial x \partial y} \right) - \frac{2}{\partial y} \left(\frac{\partial^2 \omega}{\partial x^2} \frac{\partial \omega}{\partial y} \right) \\ &= \frac{2(1+\nu)}{E} \frac{\partial^2 \tau_{xy}}{\partial x \partial y} - \left(\frac{\partial^2 \omega}{\partial x \partial y} \right)^2 - \frac{\partial \omega}{\partial x} \frac{\partial^3 \omega}{\partial x^2 \partial y} - \frac{\partial^3 \omega}{\partial x^2 \partial y} \frac{\partial \omega}{\partial y} - \frac{\partial^2 \omega}{\partial x^2} \frac{\partial^2 \omega}{\partial y^2} \end{aligned}$$

$$\begin{aligned} & \frac{\partial^2 \sigma_x}{\partial y^2} - \nu \frac{\partial^2 \sigma_y}{\partial y^2} + \frac{\partial^2 \sigma_y}{\partial x^2} - \nu \frac{\partial^2 \sigma_x}{\partial x^2} - 2(1+\nu) \frac{\partial^2 \tau_{xy}}{\partial x \partial y} \\ &= E \left[\left(\frac{\partial^2 \omega}{\partial x \partial y} \right)^2 - \frac{\partial^2 \omega}{\partial x^2} \frac{\partial^2 \omega}{\partial y^2} - \frac{1}{R} \frac{\partial^2 \omega}{\partial x^2} \right] \end{aligned}$$

$$\Delta \Delta F = E \left[\left(\frac{\partial^2 \omega}{\partial x \partial y} \right)^2 - \frac{\partial^2 \omega}{\partial x^2} \frac{\partial^2 \omega}{\partial y^2} - \frac{1}{R} \frac{\partial^2 \omega}{\partial x^2} \right] \quad (I)$$

$$\frac{Et^3}{12(1-\nu^2)} \Delta \Delta \Delta \omega + \frac{E}{R^2} \frac{\partial^4 \omega}{\partial x^4} + \Delta \Delta \left(\frac{\partial^2 F}{\partial y^2} \frac{\partial^2 \omega}{\partial x^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 \omega}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 \omega}{\partial y^2} \right) = 0 \quad (II)$$

$$\frac{w}{R} = f_1 \cos \frac{n\psi}{R} \cos \frac{m\chi}{R} + f_2$$

$$\frac{\partial w}{\partial \psi} = -f_1 n \sin \frac{n\psi}{R} \cos \frac{m\chi}{R}$$

$$\frac{\partial^2 w}{\partial \chi^2} = -f_1 \frac{m^2}{R} \cos \frac{n\psi}{R} \cos \frac{m\chi}{R}, \quad \frac{\partial^2 w}{\partial \psi^2} = -f_1 \frac{n^2}{R} \cos \frac{n\psi}{R} \cos \frac{m\chi}{R}$$

$$\frac{\partial^2 w}{\partial \chi \partial \psi} = f_1 \frac{nm}{R} \sin \frac{n\psi}{R} \sin \frac{m\chi}{R}$$

$$\Delta \Delta F = E \left[f_1^2 \frac{m^2 n^2}{R^2} \left\{ \sin^2 \frac{n\psi}{R} \sin^2 \frac{m\chi}{R} - \cos^2 \frac{n\psi}{R} \cos^2 \frac{m\chi}{R} \right\} + f_1 \frac{m^2}{R^2} \cos \frac{n\psi}{R} \cos \frac{m\chi}{R} \right]$$

$$= E \left[-\frac{1}{2} f_1^2 \frac{m^2 n^2}{R^2} \left(\cos \frac{2n\psi}{R} + \cos \frac{2m\chi}{R} \right) + f_1 \frac{m^2}{R^2} \cos \frac{n\psi}{R} \cos \frac{m\chi}{R} \right]$$

$$F = -\frac{1}{2} \sigma \psi^2 + \frac{1}{2} \alpha \chi^2 + E \left[f_1 \frac{m^2}{R^2} \frac{\cos \frac{n\psi}{R} \cos \frac{m\chi}{R}}{\left\{ \left(\frac{n}{R} \right)^2 + \left(\frac{m}{R} \right)^2 \right\}^2} \right.$$

$$\left. - \frac{1}{2} f_1^2 \frac{m^2 n^2}{R^2} \left\{ \left(\frac{R}{2n} \right)^4 \cos \frac{2n\psi}{R} + \left(\frac{R}{2m} \right)^4 \cos \frac{2m\chi}{R} \right\} \right]$$

$$\sigma_x = -\sigma + E \left[-f_1 \frac{m^2 n^2}{(n^2 + m^2)^2} \cos \frac{n\psi}{R} \cos \frac{m\chi}{R} + \frac{1}{8} f_1^2 m^2 \cos \frac{2n\psi}{R} \right]$$

$$\sigma_y = \alpha + E \left[-f_1 \frac{m^4}{(n^2 + m^2)^2} \cos \frac{n\psi}{R} \cos \frac{m\chi}{R} + \frac{1}{8} f_1^2 n^2 \cos \frac{2m\chi}{R} \right]$$

$$\frac{1}{E}(\phi_y - v\phi_x) = \frac{1}{E}(\alpha + v\sigma) + \frac{1}{f} f_1^2 \left\{ n^2 \cos \frac{2\pi x}{R} - v m^2 \cos \frac{2\pi x}{R} \right\} \quad \underline{\underline{57f}}$$

$$- f_1 m^2 \frac{m^2 - v n^2}{(n^2 + m^2)^2} \cos \frac{\pi y}{R} \cos \frac{\pi x}{R}$$

$$\frac{1}{2} \left(\frac{\partial \psi}{\partial y} \right)^2 = \frac{1}{f} f_1^2 n^2 (1 - \cos \frac{2\pi x}{R}) (1 + \cos \frac{2\pi x}{R})$$

$$= \frac{1}{f} f_1^2 n^2 \left(1 + \cos \frac{2\pi x}{R} - \cos \frac{2\pi x}{R} - \cos \frac{2\pi x}{R} \cos \frac{2\pi x}{R} \right)$$

$$\therefore \frac{\partial \psi}{\partial y} = \frac{1}{E}(\phi_y - v\phi_x) - \frac{1}{2} \left(\frac{\partial \psi}{\partial y} \right)^2 + \frac{\omega}{R}$$

$$\frac{\partial \psi}{\partial y} = \left[\frac{1}{E}(\alpha + v\sigma) - \frac{1}{f} f_1^2 n^2 + f_2 \right] + \frac{1}{f} f_1^2 (n^2 - v m^2) \cos \frac{2\pi x}{R}$$

$$+ f_1 \left\{ 1 - \frac{m^2(m^2 - v n^2)}{(n^2 + m^2)^2} \right\} \cos \frac{\pi y}{R} \cos \frac{\pi x}{R}$$

$$+ \frac{1}{f} f_1^2 n^2 \cos \frac{2\pi y}{R} \cos \frac{2\pi x}{R}$$

Put $\boxed{\frac{1}{E}(\alpha + v\sigma) - \frac{1}{f} f_1^2 n^2 + f_2 = 0} \quad \text{(IV)}$

$$\frac{\psi}{R} = \frac{1}{16} f_1^2 \frac{(n^2 - v m^2)}{n} \sin \frac{2\pi y}{R} + \frac{1}{f} \left\{ \frac{1}{n} - \frac{m^2(m^2 - v n^2)}{n(n^2 + m^2)^2} \right\} \sin \frac{\pi y}{R} \cos \frac{\pi x}{R}$$

$$+ \frac{1}{16} f_1^2 n \sin \frac{2\pi y}{R} \cos \frac{2\pi x}{R}$$

$$\frac{1}{E}(\sigma_x - \nu \sigma_y) = \frac{1}{E}(-\sigma - \nu \alpha) + \frac{1}{f} f_1^2 \left[m^2 \cos \frac{2\pi y}{R} - \nu n^2 \cos \frac{2\pi x}{R} \right] \quad \underline{\underline{579}}$$

$$- f_1 m^2 \frac{n^2 - \nu m^2}{(n^2 + m^2)^2} \cos \frac{\pi y}{R} \cos \frac{\pi x}{R}$$

$$\frac{1}{2} \left(\frac{\partial u}{\partial x} \right)^2 = \frac{1}{f} f_1^2 m^2 \left(1 + \cos \frac{2\pi y}{R} - \cos \frac{2\pi x}{R} - \cos \frac{2\pi y}{R} \cos \frac{2\pi x}{R} \right)$$

$$\frac{\partial u}{\partial x} = \left[\frac{1}{E}(-\sigma - \nu \alpha) - \frac{1}{f} f_1^2 m^2 \right] + \frac{1}{f} f_1^2 (m^2 - \nu n^2) \cos \frac{2\pi x}{R}$$

$$- f_1 \frac{m^2(n^2 - \nu m^2)}{(n^2 + m^2)^2} \cos \frac{\pi y}{R} \cos \frac{\pi x}{R} + \frac{1}{f} f_1^2 m^2 \cos \frac{2\pi y}{R} \cos \frac{2\pi x}{R}$$

$$\frac{u}{R} = \frac{x}{R} \left\{ \frac{1}{E}(-\sigma - \nu \alpha) - \frac{1}{f} f_1^2 m^2 \right\} + \frac{1}{16} f_1^2 \frac{(m^2 - \nu n^2)}{m} \sin \frac{2\pi x}{R}$$

$$- f_1 \frac{m(n^2 - \nu m^2)}{(n^2 + m^2)^2} \cos \frac{\pi y}{R} \sin \frac{\pi x}{R} + \frac{1}{16} f_1^2 m \cos \frac{2\pi y}{R} \sin \frac{2\pi x}{R}$$

The wave length in x-direction

$$\frac{m \lambda_x}{R} = 2\pi$$

$$\lambda_x = \frac{2\pi R}{m}$$

The increase in potential of σ in one length λ_x ,

$$f\sigma = \sigma \pm 2\pi R \cdot \frac{2\pi R}{m} \left\{ \frac{1}{E}(-\sigma - \nu \alpha) - \frac{1}{f} f_1^2 m^2 \right\}$$

the extensional energy = \mathcal{E}_1

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$$\mathcal{E}_1 = \frac{t}{2E} \int_0^{2\pi R} \int_{-\frac{\pi R}{m}}^{+\frac{\pi R}{m}} [(\sigma_x + \sigma_y)^2 - 2(1+\nu)(\sigma_x \sigma_y - \tau_{xy}^2)] dx dy$$

$$\sigma_x + \sigma_y = (-\sigma + \alpha) + E \left[-f_1 \frac{m^2}{n^2 + m^2} \cos \frac{n y}{R} \cos \frac{n x}{R} + \frac{1}{8} f_1^2 m^2 \cos \frac{2n y}{R} + \frac{1}{8} f_1^2 n^2 \cos \frac{2n x}{R} \right]$$

$$\mathcal{E}_{1a} = \frac{t}{2E} \left[(-\sigma + \alpha)^2 2\pi R \frac{2\pi R}{m} + E^2 \left\{ f_1^2 \frac{m^4}{(n^2 + m^2)^2} \pi R \frac{\pi R}{m} + \frac{1}{64} f_1^4 n^4 2\pi R \frac{\pi R}{m} + \frac{1}{64} f_1^2 n^4 2\pi R \frac{\pi R}{m} \right\} \right]$$

$$= \frac{t}{E} (\pi R)^2 \frac{1}{m} \left[2(-\sigma + \alpha)^2 + E^2 \left\{ \frac{1}{2} \frac{m^4}{(n^2 + m^2)^2} f_1^2 + \frac{1}{64} f_1^2 (m^4 + n^4) \right\} \right]$$

$$\boxed{\mathcal{E}_{1a} = \frac{t}{E} \frac{(\pi R)^2}{m} \left[2(-\sigma + \alpha)^2 + \frac{E^2 f_1^2}{2} \left\{ \frac{m^4}{(n^2 + m^2)^2} + \frac{f_1^2}{32} (m^4 + n^4) \right\} \right]}$$

$$\mathcal{E}_{1b} = \frac{t}{2E} \int_0^{2\pi R} \int_{-\frac{\pi R}{m}}^{+\frac{\pi R}{m}} \sigma_x \sigma_y dx dy$$

$$= \frac{t}{2E} \left\{ -\sigma \alpha 2\pi R \frac{2\pi R}{m} + E^2 f_1^2 \frac{m^6 n^2}{(n^2 + m^2)^4} \frac{(\pi R)^2}{m} \right\}$$

$$\boxed{\mathcal{E}_{1b} = \frac{t}{E} \frac{(\pi R)^2}{m} \left\{ -2\sigma \alpha + \frac{E^2 f_1^2}{2} \frac{m^6 n^2}{(n^2 + m^2)^4} \right\}}$$

$$\tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} = -E \left[\int_1 \frac{m^3 n}{(n^2 + m^2)^2} \sin \frac{\pi y}{R} \sin \frac{\pi x}{R} \right]$$

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$$G_{1c} = \frac{1}{2E} \int_0^{2\pi R} \int_{-\frac{\pi R}{m}}^{\frac{\pi R}{m}} E^2 \int_1^2 \frac{m^6 n^2}{(n^2 + m^2)^4} \sin^2 \frac{\pi y}{R} \sin^2 \frac{\pi x}{R} dx dy$$

$$G_{1c} = \frac{1}{2E} E^2 \int_1^2 \frac{m^6 n^2}{(n^2 + m^2)^4} \frac{(\pi R)^2}{m}$$

$$G_1 = \frac{1}{E} \frac{(\pi R)^2}{m} \left[2(-\sigma + \alpha)^2 + \frac{E \int_1^2}{2} \left\{ \frac{m^4}{(n^2 + m^2)^2} + \frac{1}{32} (m^4 n^4) \right\} \right. \\ \left. - 2(1+\nu) \left\{ -2\sigma\alpha + \frac{E \int_1^2}{2} \frac{m^6 n^2}{(n^2 + m^2)^4} - \frac{E \int_1^2}{2} \frac{m^6 n^2}{(n^2 + m^2)^4} \right\} \right]$$

$$G_1 = \frac{1}{E} \frac{(\pi R)^2}{m} \left[2 \left\{ (-\sigma + \alpha)^2 + 2(1+\nu) \sigma \alpha \right\} + \frac{E \int_1^2}{2} \left\{ \frac{m^4}{(n^2 + m^2)^2} + \frac{\int_1^2 (m^4 n^4)}{32} \right\} \right]$$

$$K_1 = \frac{\partial^2 \omega}{\partial x^2}$$

$$K_2 = \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial}{\partial y} \left(\frac{\nu}{R} \right)$$

$$K_{12} = \frac{\partial}{\partial x} \left(\frac{\partial \omega}{\partial y} + \frac{\nu}{R} \right)$$

$$K_1 = -\int_1 \frac{m^2}{R} \cos \frac{\pi y}{R} \cos \frac{\pi x}{R}$$

$$K_2 = -\int_1 \frac{n^2}{R} \cos \frac{\pi y}{R} \cos \frac{\pi x}{R} + \frac{1}{R} \int_1 \frac{1}{R} (n^2 - \nu m^2) \cos \frac{2\pi y}{R}$$

$$+ \int_1 \frac{1}{R} \left\{ 1 - \frac{m^2(m^2 - \nu n^2)}{(n^2 + m^2)^2} \right\} \cos \frac{\pi y}{R} \cos \frac{\pi x}{R} + \frac{1}{R} \int_1 \frac{1}{R} n^2 \cos \frac{2\pi y}{R} \cos \frac{2\pi x}{R}$$

SF2

$$K_2 = \frac{1}{R} \left[f_1 \left\{ 1 - n^2 - \frac{m^2(m^2 - \nu n^2)}{(n^2 + m^2)^2} \right\} \cos \frac{n\delta}{R} \cos \frac{m\chi}{R} + \frac{1}{\rho} f_1^2 (n^2 - \nu m^2) \cos \frac{2n\delta}{R} \right. \\ \left. + \frac{1}{\rho} f_1^2 n^2 \cos \frac{2n\delta}{R} \cos \frac{2m\chi}{R} \right]$$

$$K_2 = f_1 \frac{n m}{R} \sin \frac{n\delta}{R} \sin \frac{m\chi}{R} - f_1 \frac{n}{R} \left\{ \frac{1}{n} - \frac{m^2(m^2 - \nu n^2)}{n(n^2 + m^2)^2} \right\} \sin \frac{n\delta}{R} \sin \frac{m\chi}{R} \\ - \frac{1}{\rho} \frac{1}{R} f_1^2 m n \sin \frac{2n\delta}{R} \sin \frac{2m\chi}{R}$$

$$K_{12} = \frac{1}{R} \left[f_1 m \left\{ n - \frac{1}{n} - \frac{m^2(m^2 - \nu n^2)}{n(n^2 + m^2)^2} \right\} \sin \frac{n\delta}{R} \sin \frac{m\chi}{R} - \right. \\ \left. - \frac{1}{\rho} f_1^2 m n \sin \frac{2n\delta}{R} \sin \frac{2m\chi}{R} \right]$$

$$\mathcal{G}_2 = \frac{E t^3}{24(1-\nu^2) R^3} \left[\frac{(\pi R)^2}{m} f_1^2 \left\{ 1 - m^2 - n^2 - \frac{m^2(m^2 - \nu n^2)}{(n^2 + m^2)^2} \right\}^2 \right. \\ + \frac{2(\pi R)^2}{m} \frac{1}{64} f_1^4 (n^2 - \nu m^2)^2 + \frac{(\pi R)^2}{m} \frac{1}{64} f_1^4 n^4 \\ \left. - \frac{2(1-\nu)}{m} \left\{ - f_1^2 \frac{(\pi R)^2}{m} m^2 \left[1 - n^2 - \frac{m^2(m^2 - \nu n^2)}{(n^2 + m^2)^2} \right] - f_1^2 m^2 \left[n - \frac{1}{n} - \frac{m^2(m^2 - \nu n^2)}{n(n^2 + m^2)^2} \right] \frac{(\pi R)^2}{m} \right. \right. \\ \left. \left. - \frac{1}{64} f_1^4 m^2 n^2 \frac{(\pi R)^2}{m} \right\} \right]$$

$$\begin{aligned} \mathcal{E}_2 = & \frac{Et}{(1-\nu^2)} \frac{1}{12} \left(\frac{t}{R}\right)^2 \frac{(\pi R)^2}{m} \left[\frac{f_1^2}{2} \left\{ 1 - m^2 - n^2 - \frac{m^2(m^2 - \nu n^2)}{(n^2 + m^2)^2} \right\}^2 \right. \\ & + \frac{1}{64} f_1^4 \left\{ (n^2 - \nu m^2)^2 + \frac{n^4}{2} \right\} + (1-\nu) f_1^2 m^2 \left\{ 1 - n^2 - \frac{m^2(m^2 - \nu n^2)}{(n^2 + m^2)^2} \right\} \\ & \left. + (1-\nu) f_1^2 \left(\frac{m}{n}\right)^2 \left\{ n^2 - 1 - \frac{m^2(m^2 - \nu n^2)}{(n^2 + m^2)^2} \right\}^2 + (1-\nu) \frac{f_1^4}{64} m^2 n^2 \right] \end{aligned} \quad \underline{\underline{5f3}}$$

The total potential $\div Et \frac{(\pi R)^2}{m}$

$$\begin{aligned} = & \frac{1}{E^2} 2 \left\{ (-\sigma + \alpha)^2 + 2(1+\nu)\sigma\alpha \right\} + \frac{f_1^2}{2} \left\{ \frac{m^4}{(n^2 + m^2)^2} + f_1^2 \frac{(m^4 + n^4)}{32} \right\} \\ & + \frac{1}{12(1-\nu^2)} \left(\frac{t}{R}\right)^2 \left[\frac{f_1^2}{2} \left\{ 1 - m^2 - n^2 - \frac{m^2(m^2 - \nu n^2)}{(n^2 + m^2)^2} \right\}^2 + \frac{1}{64} f_1^4 \left\{ (n^2 - \nu m^2)^2 + \frac{n^4}{2} \right\} \right. \\ & + (1-\nu) f_1^2 m^2 \left\{ 1 - n^2 - \frac{m^2(m^2 - \nu n^2)}{(n^2 + m^2)^2} \right\} + (1-\nu) f_1^2 \left(\frac{m}{n}\right)^2 \left\{ n^2 - 1 - \frac{m^2(m^2 - \nu n^2)}{(n^2 + m^2)^2} \right\}^2 \\ & \left. + (1-\nu) \frac{f_1^4}{64} m^2 n^2 \right] - 4 \frac{\sigma}{E} \left[\frac{1}{E} (\sigma + \nu\alpha) + \frac{1}{8} f_1^2 m^2 \right] \end{aligned}$$

$$\frac{1}{E} (\alpha + \nu\sigma) = \frac{1}{8} f_1^2 n^2 - f_2$$

$$\frac{\alpha}{E} = -\nu \frac{\sigma}{E} + \frac{1}{8} f_1^2 n^2 - f_2$$

$$\frac{1}{E} (\sigma + \nu\alpha) = \frac{\sigma}{E} (1-\nu^2) + \nu \left(\frac{1}{8} f_1^2 n^2 - f_2 \right)$$

$$\frac{1}{E} (-\sigma + \alpha) = -(1+\nu) \frac{\sigma}{E} + \frac{1}{8} f_1^2 n^2 - f_2$$

$$\frac{1}{E^2} \sigma \alpha = -v \left(\frac{\sigma}{E} \right)^2 + \frac{\sigma}{E} \left(\frac{1}{f} f_1^2 n^2 - f_2 \right)$$

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hierfore

$$\begin{aligned} & 2 \left\{ \left(\frac{-\sigma + \alpha}{E} \right)^2 + 2(1+v) \frac{\sigma \alpha}{E^2} \right\} - 4 \frac{\sigma}{E} \left[\frac{\sigma + v \alpha}{E} + \frac{1}{f} f_1^2 n^2 \right] \\ &= 2 \left\{ (1+v) \left(\frac{\sigma}{E} \right)^2 - 2(1+v) \frac{\sigma}{E} \left(\frac{1}{f} f_1^2 n^2 - f_2 \right) + \left(\frac{1}{f} f_1^2 n^2 - f_2 \right)^2 \right. \\ &\quad \left. - 2(1+v) v \left(\frac{\sigma}{E} \right)^2 + 2(1+v) \frac{\sigma}{E} \left(\frac{1}{f} f_1^2 n^2 - f_2 \right) \right\} \\ &\quad - 4 \frac{\sigma}{E} \left[\frac{\sigma}{E} (1-v^2) + v \left(\frac{1}{f} f_1^2 n^2 - f_2 \right) + \frac{1}{f} f_1^2 n^2 \right] \\ &= -2 \left(\frac{\sigma}{E} \right)^2 (1-v^2) + 2 \left(\frac{1}{f} f_1^2 n^2 - f_2 \right)^2 - 4 \frac{\sigma}{E} \left\{ \frac{1}{f} f_1^2 n^2 + v \left(\frac{1}{f} f_1^2 n^2 - f_2 \right) \right\} \\ A &= -2 \left(\frac{\sigma}{E} \right)^2 (1-v^2) + 2 \left(\frac{1}{f} f_1^2 n^2 - f_2 \right)^2 - 4 \frac{\sigma}{E} \left\{ \frac{1}{f} f_1^2 (n^2 + v n^2) - v f_2 \right\} \\ &\quad - \left(\frac{1}{f} f_1^2 n^2 - f_2 \right) + \frac{\sigma}{E} v = 0 \end{aligned}$$

$$\boxed{f_2 = \frac{1}{f} f_1^2 n^2 - \frac{\sigma}{E} v}$$

$$\begin{aligned} A &= -2 \left(\frac{\sigma}{E} \right)^2 (1-v^2) + 2 \left(\frac{\sigma}{E} \right)^2 v^2 - 4 v^2 \left(\frac{\sigma}{E} \right)^2 - \frac{1}{2} \frac{\sigma}{E} f_1^2 n^2 \\ &= -2 \left(\frac{\sigma}{E} \right)^2 - \frac{1}{2} \frac{\sigma}{E} f_1^2 n^2 \end{aligned}$$

$$\frac{1}{2} \left(\frac{v}{E} \right) m^2 = \frac{1}{2} \left\{ \frac{m^4}{(n^2 + m^2)^2} + \frac{(m^2 + n^2)}{16} f_1^2 \right\} \quad \underline{\underline{5f5}}$$

$$+ \frac{1}{12(1-v^2)} \left(\frac{f_1}{R} \right)^2 \left[\frac{1}{2} \left\{ 1 - m^2 - n^2 - \frac{m^2(n^2 - v n^2)}{(n^2 + m^2)^2} \right\}^2 + \frac{1}{32} \left\{ (n^2 - v m^2)^2 + \frac{n^4}{2} \right\} f_1^2 \right]$$

$$+ (1-v) m^2 \left\{ 1 - n^2 - \frac{m^2(n^2 - v n^2)}{(n^2 + m^2)^2} \right\} + (1-v) \left(\frac{n}{n} \right)^2 \left\{ n^2 - 1 - \frac{m^2(n^2 - v n^2)}{(n^2 + m^2)^2} \right\}^2$$

$$+ (1-v) m^2 n^2 \frac{f_1^2}{32} \int$$

No Good !!!

$$\begin{aligned}
 \frac{1}{R} = & a_{00} + a_{01} \cos \frac{\pi y}{R} + a_{02} \cos \frac{2\pi y}{R} + a_{03} \cos \frac{3\pi y}{R} \\
 & + a_{10} \cos \frac{\pi x}{R} + a_{11} \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + a_{12} \cos \frac{\pi x}{R} \cos \frac{2\pi y}{R} + a_{13} \cos \frac{\pi x}{R} \cos \frac{3\pi y}{R} \\
 & + a_{20} \cos \frac{2\pi x}{R} + a_{21} \cos \frac{2\pi x}{R} \cos \frac{\pi y}{R} + a_{22} \cos \frac{2\pi x}{R} \cos \frac{2\pi y}{R} + a_{23} \cos \frac{2\pi x}{R} \cos \frac{3\pi y}{R} \\
 & + a_{30} \cos \frac{3\pi x}{R} + a_{31} \cos \frac{3\pi x}{R} \cos \frac{\pi y}{R} + a_{32} \cos \frac{3\pi x}{R} \cos \frac{2\pi y}{R} + a_{33} \cos \frac{3\pi x}{R} \cos \frac{3\pi y}{R}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{R} \frac{\partial^2 w}{\partial x^2} = & - \left(\frac{\pi}{R} \right)^2 \left\{ a_{10} \cos \frac{\pi x}{R} + a_{11} \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + a_{12} \cos \frac{\pi x}{R} \cos \frac{2\pi y}{R} + a_{13} \cos \frac{\pi x}{R} \cos \frac{3\pi y}{R} \right\} \\
 & - \left(\frac{\pi}{R} \right)^2 \left\{ 4a_{20} \cos \frac{2\pi x}{R} + 4a_{21} \cos \frac{2\pi x}{R} \cos \frac{\pi y}{R} + 4a_{22} \cos \frac{2\pi x}{R} \cos \frac{2\pi y}{R} + 4a_{23} \cos \frac{2\pi x}{R} \cos \frac{3\pi y}{R} \right\} \\
 & - \left(\frac{\pi}{R} \right)^2 \left\{ 9a_{30} \cos \frac{3\pi x}{R} + 9a_{31} \cos \frac{3\pi x}{R} \cos \frac{\pi y}{R} + 9a_{32} \cos \frac{3\pi x}{R} \cos \frac{2\pi y}{R} + 9a_{33} \cos \frac{3\pi x}{R} \cos \frac{3\pi y}{R} \right\}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{R} \frac{\partial^2 w}{\partial y^2} = & - \left(\frac{\pi}{R} \right)^2 \left\{ a_{01} \cos \frac{\pi y}{R} + 4a_{02} \cos \frac{2\pi y}{R} + 9a_{03} \cos \frac{3\pi y}{R} \right\} \\
 & - \left(\frac{\pi}{R} \right)^2 \left\{ a_{11} \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + 4a_{12} \cos \frac{\pi x}{R} \cos \frac{2\pi y}{R} + 9a_{13} \cos \frac{\pi x}{R} \cos \frac{3\pi y}{R} \right\} \\
 & - \left(\frac{\pi}{R} \right)^2 \left\{ a_{21} \cos \frac{2\pi x}{R} \cos \frac{\pi y}{R} + 4a_{22} \cos \frac{2\pi x}{R} \cos \frac{2\pi y}{R} + 9a_{23} \cos \frac{2\pi x}{R} \cos \frac{3\pi y}{R} \right\} \\
 & - \left(\frac{\pi}{R} \right)^2 \left\{ a_{31} \cos \frac{3\pi x}{R} \cos \frac{\pi y}{R} + 4a_{32} \cos \frac{3\pi x}{R} \cos \frac{2\pi y}{R} + 9a_{33} \cos \frac{3\pi x}{R} \cos \frac{3\pi y}{R} \right\}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{R} \frac{\partial^2 w}{\partial x \partial y} = & \frac{\pi^2}{R^2} \left\{ a_{11} \sin \frac{\pi x}{R} \sin \frac{\pi y}{R} + 2a_{12} \sin \frac{\pi x}{R} \sin \frac{2\pi y}{R} + 3a_{13} \sin \frac{\pi x}{R} \sin \frac{3\pi y}{R} \right. \\
 & + 2a_{21} \sin \frac{2\pi x}{R} \sin \frac{\pi y}{R} + 4a_{22} \sin \frac{2\pi x}{R} \sin \frac{2\pi y}{R} + 6a_{23} \sin \frac{2\pi x}{R} \sin \frac{3\pi y}{R} \\
 & \left. + 3a_{31} \sin \frac{3\pi x}{R} \sin \frac{\pi y}{R} + 6a_{32} \sin \frac{3\pi x}{R} \sin \frac{2\pi y}{R} + 9a_{33} \sin \frac{3\pi x}{R} \sin \frac{3\pi y}{R} \right\}
 \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 w}{\partial x \partial y} = \frac{m\pi}{R} & \left[\sin \frac{m\pi}{R} \left\{ a_{11} \sin \frac{n\pi}{R} + 2a_{12} \sin \frac{2n\pi}{R} + 3a_{13} \sin \frac{3n\pi}{R} \right\} \right. \\ & + 2 \sin \frac{2m\pi}{R} \left\{ a_{21} \sin \frac{n\pi}{R} + 2a_{22} \sin \frac{2n\pi}{R} + 3a_{23} \sin \frac{3n\pi}{R} \right\} \\ & \left. + 3 \sin \frac{3m\pi}{R} \left\{ a_{31} \sin \frac{n\pi}{R} + 2a_{32} \sin \frac{2n\pi}{R} + 3a_{33} \sin \frac{3n\pi}{R} \right\} \right] \quad \underline{\underline{5d2}} \end{aligned}$$

$$\begin{aligned} \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 = \frac{m^2 \pi^2}{R^2} & \left[\frac{1}{2} (1 - \cos \frac{2m\pi}{R}) \left\{ a_{11} \sin \frac{n\pi}{R} + 2a_{12} \sin \frac{2n\pi}{R} + 3a_{13} \sin \frac{3n\pi}{R} \right\}^2 \right. \\ & + 2 (1 - \cos \frac{4m\pi}{R}) \left\{ a_{21} \sin \frac{n\pi}{R} + 2a_{22} \sin \frac{2n\pi}{R} + 3a_{23} \sin \frac{3n\pi}{R} \right\}^2 \\ & + \frac{9}{2} (1 - \cos \frac{6m\pi}{R}) \left\{ a_{31} \sin \frac{n\pi}{R} + 2a_{32} \sin \frac{2n\pi}{R} + 3a_{33} \sin \frac{3n\pi}{R} \right\}^2 \\ & + 2 \left(\cos \frac{m\pi}{R} - \cos \frac{3m\pi}{R} \right) \left\{ a_{11} \sin \theta + 2a_{12} \sin 2\theta + 3a_{13} \sin 3\theta \right\} \left\{ a_{21} \sin \theta + 2a_{22} \sin 2\theta + 3a_{23} \sin 3\theta \right\} \\ & + 6 \left(\cos \frac{m\pi}{R} - \cos \frac{5m\pi}{R} \right) \left\{ a_{21} \sin \theta + 2a_{22} \sin 2\theta + 3a_{23} \sin 3\theta \right\} \left\{ a_{31} \sin \theta + 2a_{32} \sin 2\theta + 3a_{33} \sin 3\theta \right\} \\ & + 3 \left(\cos \frac{2m\pi}{R} - \cos \frac{4m\pi}{R} \right) \left\{ a_{31} \sin \theta + 2a_{32} \sin 2\theta + 3a_{33} \sin 3\theta \right\} \left\{ a_{11} \sin \theta + 2a_{12} \sin 2\theta + 3a_{13} \sin 3\theta \right\} \right] \end{aligned}$$

$$\begin{aligned} & (a_{11} \sin \theta + 2a_{12} \sin 2\theta + 3a_{13} \sin 3\theta)^2 \\ & = a_{11}^2 \sin^2 \theta + 4a_{12}^2 \sin^2 2\theta + 9a_{13}^2 \sin^2 3\theta + 4a_{11}a_{12} \sin \theta \sin 2\theta + 12a_{12}a_{13} \sin 2\theta \sin 3\theta \\ & \quad + 6a_{13}a_{11} \sin 3\theta \sin \theta \\ & = \frac{1}{2} a_{11}^2 (1 - \cos 2\theta) + 2a_{12}^2 (1 - \cos 4\theta) + \frac{9}{2} a_{13}^2 (1 - \cos 6\theta) \\ & \quad + 2a_{11}a_{12} (\cos \theta - \cos 3\theta) + 6a_{12}a_{13} (\cos \theta - \cos 5\theta) + 3a_{13}a_{11} (\cos 2\theta - \cos 4\theta) \end{aligned}$$

$$\begin{aligned}
 & (a_{11} \sin \delta + 2a_{12} \sin 2\delta + 3a_{13} \sin 3\delta)^2 \\
 &= \left(\frac{1}{2} a_{11}^2 + 2a_{12}^2 + \frac{9}{2} a_{13}^2 \right) + (2a_{11}a_{12} + 6a_{12}a_{13}) \cos \delta + \left(3a_{13}a_{11} - \frac{1}{2} a_{11}^2 \right) \cos 2\delta - 2a_{11}a_{12} \cos 3\delta \\
 &\quad - (2a_{12}^2 + 3a_{13}a_{11}) \cos 4\delta - 6a_{12}a_{13} \cos 5\delta - \frac{9}{2} a_{13}^2 \cos 6\delta
 \end{aligned}$$

$$\begin{aligned}
 \left(\frac{\partial \bar{u}}{\partial y} \right)^2 &= \frac{m^2 a^2}{R^2} \left[\frac{1}{2} (1 - \cos 2\varphi) \left\{ \left(\frac{1}{2} a_{11}^2 + 2a_{12}^2 + \frac{9}{2} a_{13}^2 \right) + (2a_{11}a_{12} + 6a_{12}a_{13}) \cos \delta + (3a_{13}a_{11} - \frac{1}{2} a_{11}^2) \cos 2\delta \right. \right. \\
 &\quad \left. \left. - 2a_{11}a_{12} \cos 3\delta - (2a_{12}^2 + 3a_{13}a_{11}) \cos 4\delta - 6a_{12}a_{13} \cos 5\delta - \frac{9}{2} a_{13}^2 \cos 6\delta \right\} \right. \\
 &\quad \left. + 2(1 - \cos 4\varphi) \left\{ \left(\frac{1}{2} a_{21}^2 + 2a_{22}^2 + \frac{9}{2} a_{23}^2 \right) + (2a_{21}a_{22} + 6a_{22}a_{23}) \cos \delta + (3a_{23}a_{21} - \frac{1}{2} a_{21}^2) \cos 2\delta \right. \right. \\
 &\quad \left. \left. - 2a_{21}a_{22} \cos 3\delta - (2a_{22}^2 + 3a_{23}a_{21}) \cos 4\delta - 6a_{22}a_{23} \cos 5\delta - \frac{9}{2} a_{23}^2 \cos 6\delta \right\} \right. \\
 &\quad \left. + \frac{9}{2} (1 - \cos 6\varphi) \left\{ \left(\frac{1}{2} a_{31}^2 + 2a_{32}^2 + \frac{9}{2} a_{33}^2 \right) + (2a_{31}a_{32} + 6a_{32}a_{33}) \cos \delta + (3a_{33}a_{31} - \frac{1}{2} a_{31}^2) \cos 2\delta \right. \right. \\
 &\quad \left. \left. - 2a_{31}a_{32} \cos 3\delta - (2a_{32}^2 + 3a_{33}a_{31}) \cos 4\delta - 6a_{32}a_{33} \cos 5\delta - \frac{9}{2} a_{33}^2 \cos 6\delta \right\} \right]
 \end{aligned}$$

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$$\frac{1}{R} \omega = f_1 + f_2 \cos \frac{mX}{R} \cos \frac{mY}{R} + f_3 \cos \frac{2mX}{R} + f_4 \cos \frac{2mY}{R} \quad \underline{529}$$

$$\frac{1}{R} \frac{\partial^2 \omega}{\partial X^2} = - \left(\frac{m}{R} \right)^2 \left[f_2 \cos \frac{mX}{R} \cos \frac{mY}{R} + 4f_3 \cos \frac{2mX}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 \omega}{\partial Y^2} = - \left(\frac{m}{R} \right)^2 \left[f_2 \cos \frac{mX}{R} \cos \frac{mY}{R} + 4f_4 \cos \frac{2mY}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 \omega}{\partial X \partial Y} = + \left(\frac{m}{R} \right)^2 \left[f_2 \sin \frac{mX}{R} \sin \frac{mY}{R} \right]$$

$$\text{Thus } \left(\frac{1}{R} \frac{\partial^2 \omega}{\partial X \partial Y} \right)^2 = \left(\frac{m}{R} \right)^4 \left[f_2^2 \sin^2 \frac{mX}{R} \sin^2 \frac{mY}{R} \right]$$

$$\begin{aligned} \left(\frac{1}{R} \frac{\partial^2 \omega}{\partial X^2} \right) \left(\frac{1}{R} \frac{\partial^2 \omega}{\partial Y^2} \right) &= \left(\frac{m}{R} \right)^4 \left[f_2^2 \cos^2 \frac{mX}{R} \cos^2 \frac{mY}{R} + 2f_2 f_3 \left(\cos \frac{3mX}{R} + \cos \frac{mX}{R} \right) \cos \frac{mY}{R} \right. \\ &\quad \left. + 2f_2 f_4 \cos \frac{mX}{R} \left(\cos \frac{3mY}{R} + \cos \frac{mY}{R} \right) + 16f_3 f_4 \cos \frac{2mX}{R} \cos \frac{2mY}{R} \right] \end{aligned}$$

$$\Delta \Delta F = \frac{m^2}{R^2} E \left[m^2 f_2^2 \left\{ \sin^2 \frac{mX}{R} \sin^2 \frac{mY}{R} - \cos^2 \frac{mX}{R} \cos^2 \frac{mY}{R} \right\} \right.$$

$$\left. - m^2 \left\{ 2f_2 f_3 \left(\cos \frac{3mX}{R} + \cos \frac{mX}{R} \right) \cos \frac{mY}{R} + 2f_2 f_4 \cos \frac{mX}{R} \left(\cos \frac{3mY}{R} + \cos \frac{mY}{R} \right) \right. \right.$$

$$\left. + 16f_3 f_4 \cos \frac{2mX}{R} \cos \frac{2mY}{R} \right\} + f_2 \cos \frac{mX}{R} \cos \frac{mY}{R} + 4f_3 \cos \frac{2mX}{R} \right]$$

$$\begin{aligned} &= \left(\frac{m}{R} \right)^2 E \left[-\frac{m^2}{2} f_2^2 \cos \frac{2mX}{R} - \frac{m^2}{2} f_2^2 \cos \frac{2mY}{R} - 16m^2 f_3 f_4 \cos \frac{2mX}{R} \cos \frac{2mY}{R} \right. \\ &\quad - \left(2m^2 f_2 f_3 + 2m^2 f_2 f_4 - f_2 \right) \cos \frac{mX}{R} \cos \frac{mY}{R} \\ &\quad \left. - 2m^2 f_2 f_3 \cos \frac{3mX}{R} \cos \frac{mY}{R} - 2m^2 f_2 f_4 \cos \frac{mX}{R} \cos \frac{3mY}{R} + 4f_3 \cos \frac{2mX}{R} \right] \end{aligned}$$

$$\Delta F = \left(\frac{m}{R}\right)^2 E \left[f_2 (1 - 2m^2 f_3 - 2m^2 f_4) \cos \frac{mX}{R} \cos \frac{mY}{R} \right. \quad \underline{\underline{590}}$$

$$+ (4f_3^2 - \frac{m^2}{2} f_2^2) \cos \frac{2mX}{R} - \frac{m^2}{2} f_2^2 \cos \frac{2mY}{R} - 16m^2 f_3 f_4 \cos \frac{2mX}{R} \cos \frac{2mY}{R} \\ \left. - 2m^2 f_2 f_3 \cos \frac{3mX}{R} \cos \frac{mY}{R} - 2m^2 f_2 f_4 \cos \frac{mX}{R} \cos \frac{3mY}{R} \right]$$

$$F = \frac{E}{\left(\frac{m}{R}\right)^2} \left[\frac{1}{4} f_2 (1 - 2m^2 f_3 - 2m^2 f_4) \cos \frac{mX}{R} \cos \frac{mY}{R} \right. \\ + \frac{1}{4} \frac{1}{4} (4f_3^2 - \frac{m^2}{2} f_2^2) \cos \frac{2mX}{R} - \frac{1}{4} \frac{m^2}{8} f_2^2 \cos \frac{2mY}{R} - \frac{1}{4} m^2 f_3 f_4 \cos \frac{2mX}{R} \cos \frac{2mY}{R} \\ \left. - \frac{1}{50} m^2 f_2 f_3 \cos \frac{3mX}{R} \cos \frac{mY}{R} - \frac{1}{50} m^2 f_2 f_4 \cos \frac{mX}{R} \cos \frac{3mY}{R} \right]$$

$$\sigma_x = E \left[-\frac{1}{4} f_2 (1 - 2m^2 f_3 - 2m^2 f_4) \cos \frac{mX}{R} \cos \frac{mY}{R} \right. \\ + \frac{m^2}{8} f_2^2 \cos \frac{2mX}{R} + m^2 f_3 f_4 \cos \frac{2mX}{R} \cos \frac{2mY}{R} + \frac{1}{50} m^2 f_2 f_3 \cos \frac{3mX}{R} \cos \frac{mY}{R} \\ \left. + \frac{9}{50} m^2 f_2 f_4 \cos \frac{mX}{R} \cos \frac{3mY}{R} \right] - \sigma$$

$$\sigma_y = E \left[-\frac{1}{4} f_2 (1 - 2m^2 f_3 - 2m^2 f_4) \cos \frac{mX}{R} \cos \frac{mY}{R} \right. \\ + \left(\frac{m^2}{8} f_2^2 - f_3 \right) \cos \frac{2mX}{R} + m^2 f_3 f_4 \cos \frac{2mX}{R} \cos \frac{2mY}{R} + \frac{9}{50} m^2 f_2 f_3 \cos \frac{3mX}{R} \cos \frac{mY}{R} \\ \left. + \frac{1}{50} m^2 f_2 f_4 \cos \frac{mX}{R} \cos \frac{3mY}{R} \right] + \alpha$$

$$\begin{aligned}
 \frac{1}{E}(\bar{v}_y - v_x) &= -\frac{1}{4}f_2(1-\nu)(1-2m^2f_3-2m^2f_4)\cos\frac{m\pi}{R}\cos\frac{m\pi}{R} \\
 &+ \left(\frac{m^2f_2^2}{f}f_3\right)\cos\frac{2m\pi}{R} - 4\frac{m^2}{f}f_2^2\cos\frac{2m\pi}{R} + (1-\nu)m^2f_3f_4\cos\frac{2m\pi}{R}\cos\frac{2m\pi}{R} \\
 &+ \frac{1}{50}(9-\nu)m^2f_2f_3\cos\frac{3m\pi}{R}\cos\frac{m\pi}{R} + \frac{1}{50}(1-9\nu)m^2f_2f_4\cos\frac{m\pi}{R}\cos\frac{3m\pi}{R} \\
 &+ \frac{1}{E}(d+4\nu)
 \end{aligned}$$

$$\begin{aligned}
 -\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 &= -\frac{m^2}{2}\left\{f_2\cos\frac{m\pi}{R}\sin\frac{m\pi}{R} + 2f_4\sin\frac{2m\pi}{R}\right\}^2 \\
 &= -m^2\left\{\frac{1}{f}f_2^2\left(1+\cos\frac{2m\pi}{R}\right)\left(1-\cos\frac{2m\pi}{R}\right) + f_2f_4\cos\frac{m\pi}{R}\left(\cos\frac{m\pi}{R} - \cos\frac{3m\pi}{R}\right) \right. \\
 &\quad \left. + f_4^2\left(1-\cos\frac{4m\pi}{R}\right)\right\}
 \end{aligned}$$

$$\begin{aligned}
 -\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 &= \left(-\frac{m^2}{f}f_2^2 - m^2f_4^2\right) - \frac{m^2}{f}f_2^2\cos\frac{2m\pi}{R} + \frac{m^2}{f}f_2^2\cos\frac{2m\pi}{R} \\
 &+ \frac{m^2}{f}f_2^2\cos\frac{2m\pi}{R}\cos\frac{2m\pi}{R} - m^2f_2f_4\cos\frac{m\pi}{R}\cos\frac{m\pi}{R} + m^2f_2f_4\cos\frac{m\pi}{R}\cos\frac{3m\pi}{R} \\
 &+ m^2f_4^2\cos\frac{4m\pi}{R}
 \end{aligned}$$

$$\frac{w}{R} = f_1 + f_2\cos\frac{m\pi}{R}\cos\frac{m\pi}{R} + f_3\cos\frac{2m\pi}{R} + f_4\cos\frac{2m\pi}{R}$$

$$\frac{\partial v}{\partial y} = \left\{ \frac{1}{E} (\alpha + \nu \sigma) - m^2 \left(\frac{1}{\rho} f_2^2 + f_4^2 \right) + f_1 \right\} + \dots$$

$$\therefore \boxed{f_1 = m^2 \left(\frac{1}{\rho} f_2^2 + f_4^2 \right) - \frac{1}{E} (\alpha + \nu \sigma)}$$

$$\begin{aligned} T_{xy} = E & \left[-\frac{1}{4} f_2 (1 - 2m^2 f_3 - 2m^2 f_4) \sin \frac{m\lambda}{R} \sin \frac{m\mu}{R} \right. \\ & + m^2 f_3 f_4 \sin \frac{2m\lambda}{R} \sin \frac{2m\mu}{R} + \frac{3}{50} m^2 f_2 f_3 \sin \frac{3m\lambda}{R} \sin \frac{m\mu}{R} \\ & \left. + \frac{3}{50} m^2 f_2 f_4 \sin \frac{m\lambda}{R} \sin \frac{3m\mu}{R} \right] \end{aligned}$$

$$\begin{aligned} \sigma_x + \sigma_y = E & \left[-\frac{1}{2} f_2 (1 - 2m^2 f_3 - 2m^2 f_4) \cos \frac{m\lambda}{R} \cos \frac{m\mu}{R} \right. \\ & + \left(\frac{m^2}{\rho} f_2^2 - f_3 \right) \cos \frac{2m\lambda}{R} + \frac{m^2}{\rho} f_2^2 \cos \frac{2m\mu}{R} + 2m^2 f_3 f_4 \cos \frac{2m\lambda}{R} \cos \frac{2m\mu}{R} \\ & \left. + \frac{1}{5} m^2 f_2 f_3 \cos \frac{3m\lambda}{R} \cos \frac{m\mu}{R} + \frac{1}{5} m^2 f_2 f_4 \cos \frac{m\lambda}{R} \cos \frac{3m\mu}{R} \right] + (\alpha - \sigma) \end{aligned}$$

$$\begin{aligned} & \frac{1}{R} \frac{1}{2E} \iint (\sigma_x + \sigma_y)^2 dx dy \\ = & \left(\frac{1}{R} \right) \frac{E}{2} \left[\frac{1}{4} f_2^2 (1 - 2m^2 f_3 - 2m^2 f_4)^2 + 2 \left(f_3 - \frac{m^2}{\rho} f_2^2 \right)^2 + 2 \left(\frac{m^2}{\rho} f_2^2 \right)^2 \right. \\ & \left. + (2m^2 f_3 f_4)^2 + \frac{1}{25} m^4 f_2^2 (f_3^2 + f_4^2) + 4 \left(\frac{\alpha}{E} - \frac{\sigma}{E} \right)^2 \right] \end{aligned}$$

$$\frac{1}{R} \frac{\hbar}{2E} \iint (\sigma_x \sigma_y) dx dy$$

$$\sim \left(\frac{\hbar}{R}\right) \frac{E}{2} \left[\frac{1}{16} f_2^2 (1 - 2m^2 f_3 - 2m^2 f_4)^2 + \frac{9}{2500} m^4 f_2^2 f_3^2 + \frac{9}{2500} m^4 f_2^2 f_4^2 - \frac{4\sigma\alpha}{E} \right] + m^4 f_3 f_4$$

$$\frac{1}{R} \frac{\hbar}{2E} \iint \tau_{xy}^2 dx dy$$

$$\sim \left(\frac{\hbar}{R}\right) \frac{E}{2} \left[\frac{1}{16} f_2^2 (1 - 2m^2 f_3 - 2m^2 f_4)^2 + m^4 f_3 f_4 + \frac{9}{2500} m^4 f_2^2 (f_3^2 + f_4^2) \right]$$

hence the extentional energy

$$\begin{aligned} & \approx \frac{1}{4} f_2^2 (1 - 2m^2 f_3 - 2m^2 f_4)^2 + 2 \left(f_3 - \frac{m^2}{8} f_2^2 \right)^2 + 2 \left(\frac{m^2}{8} f_2^2 \right)^2 \\ & \quad + 4m^4 f_3^2 f_4^2 + \frac{1}{25} m^4 f_2^2 (f_3^2 + f_4^2) + 4 \left(\frac{\alpha}{E} - \frac{\sigma}{E} \right)^2 + \frac{8(1+\nu)\sigma\alpha}{E^2} \\ & = \frac{1}{4} f_2^2 \left(1 + 4m^4 f_3^2 + 4m^4 f_4^2 - 4m^2 f_3 - 4m^2 f_4 + 8m^4 f_3 f_4 \right) \\ & \quad + 2 \left(f_3^2 - \frac{m^2}{4} f_3 f_2^2 + \frac{m^4}{64} f_2^4 \right) + \frac{m^4}{32} f_2^4 + 4m^4 f_3^2 f_4^2 \\ & \quad + \frac{1}{25} m^4 f_2^2 (f_3^2 + f_4^2) + 4 \left(\frac{\alpha^2}{E^2} + \frac{\sigma^2}{E^2} \right) + \frac{8\nu}{E^2} \sigma\alpha \\ & = f_2^2 \left(1 + \frac{26}{25} m^4 f_3^2 + \frac{26}{25} m^4 f_4^2 - \frac{3}{2} m^2 f_3 - m^2 f_4 + 2m^4 f_3 f_4 + \frac{m^4}{16} f_2^2 \right) \\ & \quad + 2f_3^2 (1 + 2m^4 f_4^2) + 4 \left[\left(\frac{\alpha}{E} \right)^2 + \left(\frac{\sigma}{E} \right)^2 \right] + 8\nu \left(\frac{\sigma}{E} \right) \left(\frac{\alpha}{E} \right) \end{aligned}$$

$$K_x = \frac{\partial^2 \psi}{\partial x^2} = -\frac{m^2}{R} \left[\frac{1}{2} \cos \frac{mx}{R} \cos \frac{my}{R} + 4f_3 \cos \frac{2mx}{R} \right]$$

$$K_y = \frac{\partial^2 \psi}{\partial y^2} = -\frac{m^2}{R} \left[\frac{1}{2} \cos \frac{mx}{R} \cos \frac{my}{R} + 4f_4 \cos \frac{2my}{R} \right]$$

$$K_{xy} = \frac{\partial^2 \psi}{\partial x \partial y} = \frac{m^2}{R} \frac{1}{2} \sin \frac{mx}{R} \sin \frac{my}{R}$$

Bending

$$\frac{1}{3} \left(\frac{1}{R} \right)^2 \frac{1}{(1-\nu^2)} m^4 \left[\frac{1}{2} + 8f_3^2 + 8f_4^2 \right] = \text{External Energy}$$

$$\frac{1}{E} (\sigma_x - \nu \sigma_y) = -\frac{1}{E} (0 + \nu \alpha) + \dots$$

$$-\frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 = - \left(\frac{m^2}{8} f_2^2 + m^2 f_4^2 \right) + \dots$$

$$\text{Increase in Potential energy} = -8 \left(\frac{\sigma}{E} \right) \left[\left(\frac{\sigma}{E} + \nu \frac{\alpha}{E} \right) + m^2 \left(\frac{1}{8} f_2^2 + f_4^2 \right) \right]$$

$$\frac{\alpha}{E} = m^2 \left(\frac{1}{8} f_2^2 + f_4^2 \right) - f_1 - \nu \frac{\sigma}{E}$$

$$\frac{\sigma}{E} + \nu \frac{\alpha}{E} = (1-\nu^2) \frac{\sigma}{E} + \nu m^2 \left(\frac{1}{8} f_2^2 + f_4^2 \right) - \nu f_1$$

$$= -8 \frac{\sigma}{E} \left[(1-\nu^2) \frac{\sigma}{E} + (1+\nu) m^2 \left(\frac{1}{8} f_2^2 + f_4^2 \right) - \nu f_1 \right]$$

$$4 \left[\left(\frac{K}{E} \right)^2 + \left(\frac{\sigma}{E} \right)^2 + 2\nu \left(\frac{\sigma}{E} \right) \left(\frac{K}{E} \right) \right]$$

$$= 4 \left[m^4 \left(\frac{1}{8} l_2^2 + l_4^2 \right)^2 + l_1^2 + \cancel{4 \left(\frac{\sigma}{E} \right)^2} - 2m^2 l_1 \left(\frac{1}{8} l_2^2 + l_4^2 \right) - \cancel{2\nu \frac{\sigma}{E} m^2 \left(\frac{1}{8} l_2^2 + l_4^2 \right)} \right. \\ \left. + \cancel{2\nu l_1 \frac{\sigma}{E}} + 2\nu m^2 \frac{\sigma}{E} \left(\frac{1}{8} l_2^2 + l_4^2 \right) - \cancel{2\nu l_1 \frac{\sigma}{E}} - \cancel{2\nu \left(\frac{\sigma}{E} \right)^2} \right]$$

$$= 4 \left[m^4 \left(\frac{1}{8} l_2^2 + l_4^2 \right)^2 + l_1^2 - 2m^2 l_1 \left(\frac{1}{8} l_2^2 + l_4^2 \right) + (1-\nu) \left(\frac{\sigma}{E} \right)^2 \right]$$

$$\left. \begin{aligned} & - 4(1-\nu) \left(\frac{\sigma}{E} \right)^2 - 8(1+\nu) \frac{\sigma}{E} m^2 \left(\frac{1}{8} l_2^2 + l_4^2 \right) + 8\nu \left(\frac{\sigma}{E} \right) l_1 \\ & + 4m^4 \left(\frac{1}{8} l_2^2 + l_4^2 \right)^2 + 4l_1^2 - 8m^2 l_1 \left(\frac{1}{8} l_2^2 + l_4^2 \right) \end{aligned} \right\}$$

$$8\nu \frac{\sigma}{E} + 8l_1 - 8m^2 \left(\frac{1}{8} l_2^2 + l_4^2 \right) = 0 \quad \left. \vphantom{\frac{\sigma}{E}} \right\} \text{Min.}$$

$$\boxed{l_1 = m^2 \left(\frac{1}{8} l_2^2 + l_4^2 \right) - \nu \frac{\sigma}{E}}$$

$$- 4(1-\nu) \left(\frac{\sigma}{E} \right)^2 - 8(1+\nu) \frac{\sigma}{E} m^2 \left(\frac{1}{8} l_2^2 + l_4^2 \right) + \cancel{8\nu \frac{\sigma}{E} m^2 \left(\frac{1}{8} l_2^2 + l_4^2 \right)}$$

$$- 8\nu \left(\frac{\sigma}{E} \right)^2 + \cancel{4m^2 \left(\frac{1}{8} l_2^2 + l_4^2 \right)^2} + \cancel{4m^4 \left(\frac{1}{8} l_2^2 + l_4^2 \right)^2} + 4\nu \left(\frac{\sigma}{E} \right)^2$$

$$- \cancel{8m^2 \frac{\sigma}{E} \nu \left(\frac{1}{8} l_2^2 + l_4^2 \right)} - \cancel{8m^4 \left(\frac{1}{8} l_2^2 + l_4^2 \right)^2} + \cancel{8m^2 \frac{\sigma}{E} \nu \left(\frac{1}{8} l_2^2 + l_4^2 \right)}$$

$$W = -4\left(\frac{E}{2}\right)^2 - 8\left(\frac{E}{2}\right)m^2\left(\frac{1}{8}f_2^2 + f_4^2\right) + \frac{f_2^2}{2}\left(1 + \frac{26}{25}m^4f_3^2 + \frac{26}{25}m^4f_4^2 - \frac{3}{2}m^2f_3 - m^2f_4 + 2m^4f_3f_4 + \frac{26}{15}f_4^2\right) + 2f_3^2(1 + 2m^4f_4^2) + \frac{1}{3}\left(\frac{1}{R}\right)^2\frac{1}{(1-v^2)}m^4(f_2^2 + 8f_3^2 + 8f_4^2)$$

$$\frac{\partial W}{\partial f_2} = 0$$

$$\left(\frac{E}{2}\right)m^2 = 1 + \frac{26}{25}m^4f_3^2 + \frac{26}{25}m^4f_4^2 - \frac{3}{2}m^2f_3 - m^2f_4 + 2m^4f_3f_4 + \frac{m^4}{8}f_2^2 + \frac{1}{3}\left(\frac{1}{R}\right)^2\frac{m^4}{1-v^2}$$

$$0 = f_2^2\left(\frac{52}{25}m^4f_3 - \frac{3}{2} + 2m^2f_4\right) + \frac{1}{3}\left(\frac{1}{R}\right)^2\frac{1}{(1-v^2)}m^216f_3 + 4f_3\left(1 + 2m^4f_4^2\right)$$

$$\frac{\partial W}{\partial f_4} = 0$$

$$2f_4\left(\frac{E}{2}\right) = -f_2^2\left(\frac{52}{25}m^2f_4 - 1 + 2m^2f_3\right) + 2f_3^2 \cdot 4m^2f_4 + \frac{1}{3}\left(\frac{1}{R}\right)^2\frac{m^2}{1-v^2}16f_4$$

Put $\frac{\sigma}{E}m^2 = \lambda$, and $f_2m^2 = \alpha$, $f_3m^2 = \beta$, $f_4m^2 = \gamma$, $\frac{1}{3}\left(\frac{1}{R}\right)^2\frac{m^4}{1-v^2} = \odot$

$$\lambda = 1 + \frac{26}{25}\beta^2 + \frac{26}{25}\gamma^2 - \frac{3}{2}\beta - \gamma + 2\beta\gamma + \frac{1}{8}\alpha^2 + \odot$$

$$0 = \alpha^2\left(\frac{52}{25}\beta - \frac{3}{2} + 2\gamma\right) + 4\beta(1 + 2\gamma^2) + 16\odot\beta$$

$$16\beta\lambda = \alpha^2\left(\frac{52}{25}\gamma - 1 + 2\beta\right) + 8\beta^2\gamma + 16\odot\gamma$$

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$$a^2 = 8 \left[\lambda - 1 - \frac{26}{25}\beta^2 - \frac{26}{25}\gamma^2 + \frac{3}{2}\beta + \gamma - 2\beta\gamma - 0 \right]$$

$$0 = 8 \left[\lambda - 1 - \frac{26}{25}(\beta^2 + \gamma^2) + \frac{3}{2}\beta + \gamma - 2\beta\gamma - 0 \right] \left(\frac{52}{26}\beta - \frac{3}{2} + 2\gamma \right) + 4\beta(1+2\gamma^2) + 160\beta$$

$$1\lambda = \frac{1}{2} \left[\lambda - 1 - \frac{26}{25}(\beta^2 + \gamma^2) + \frac{3}{2}\beta + \gamma - 2\beta\gamma - 0 \right] \left(\frac{52}{26}\gamma - 1 + 2\beta \right) + \frac{1}{2}\beta^2\gamma + 0\gamma$$

$$\frac{-\beta(1+2\gamma^2) - 40\beta}{48\lambda - 2\beta^2\gamma - 40\gamma} = \frac{\frac{52}{25}\beta - \frac{3}{2} + 2\gamma}{\frac{52}{25}\gamma - 1 + 2\beta}$$

$$\therefore \left[\beta(1+2\gamma^2) + 40\beta \right] \left[\frac{52}{25}\gamma - 1 + 2\beta \right] - \left[48\lambda - 2\beta^2\gamma - 40\gamma \right] \left(\frac{52}{25}\beta - \frac{3}{2} + 2\gamma \right) = 0$$

○ Known find λ

$$8\lambda\gamma\left(\frac{52}{25}\beta - \frac{3}{2} + 2\gamma\right) - 8\gamma\left[1 + \frac{24}{25}(\beta^2 + \gamma^2) - \frac{3}{2}\beta - \gamma + 2\beta\gamma + 0\right]\left(\frac{52}{25}\beta - \frac{3}{2} + 2\gamma\right) + 4\beta\gamma(1 + 2\gamma^2) + 16\beta\gamma = 0$$

$$-8\lambda\gamma\left(\frac{52}{25}\beta - \frac{3}{2} + 2\gamma\right) + 4(\beta^2\gamma + 2\beta\gamma)\left(\frac{52}{25}\beta - \frac{3}{2} + 2\gamma\right) + 2\left[\beta(1 + 2\gamma^2) + 4\beta\gamma\right]\left(\frac{52}{25}\gamma - 1 + 2\beta\right) = 0$$

$$\gamma\left(\frac{52}{25}\beta - \frac{3}{2} + 2\gamma\right)\left[4\beta^2 + 8\beta - 1 - \frac{24}{25}(\beta^2 + \gamma^2) + \frac{3}{2}\beta + \gamma - 2\beta\gamma - 0\right] + 2\left[\beta(1 + 2\gamma^2) + 4\beta\gamma\right]\left[2\gamma + \frac{52}{25}\gamma - 1 + 2\beta\right] = 0$$

$$\gamma\left(\frac{52}{25}\beta - \frac{3}{2} + 2\gamma\right)\left(\frac{14}{25}\beta^2 - \frac{24}{25}\gamma^2 - 1 + \frac{3}{2}\beta + \gamma - 2\beta\gamma + 10\right) + 2\beta\left[(1 + 2\gamma^2) + 4\beta\gamma\right]\left[\frac{102}{25}\gamma - 1 + 2\beta\right] = 0.$$

$$y \left[\frac{52}{25} \beta + (2y - \frac{3}{2}) \right] \left[\frac{24}{25} \beta^2 + (\frac{3}{2} - 2y) \beta + (y + 10 - \frac{26}{25} y^2 - 1) \right]$$

$$+ 2\beta \left[(1 + 2y^2) + 40 \right] \left[2\beta + (\frac{102}{25} y - 1) \right] = 0$$

$$\begin{array}{r} 24 \\ 52 \\ \hline 148 \\ 320 \end{array}$$

$$y \left[\frac{3848}{625} \beta^3 + \left[\frac{24}{25} (2y - \frac{3}{2}) + \frac{52}{25} (\frac{3}{2} - 2y) \right] \beta^2 + \left[\frac{52}{25} (y + 10 - \frac{26}{25} y^2 - 1) + (2y - \frac{3}{2}) (\frac{3}{2} - 2y) \right] \beta \right.$$

$$\left. + (2y - \frac{3}{2}) (y + 10 - \frac{26}{25} y^2 - 1) \right]$$

$$+ \left[(1 + 2y^2) + 40 \right] \left[4\beta^2 + 2(\frac{102}{25} y - 1) \beta \right] = 0.$$

$$\begin{array}{r} 24 \\ 52 \\ \hline 76 \end{array}$$

$$\left[\frac{3848}{625} y \right] \beta^3 + \left[\frac{24}{25} y (2y - \frac{3}{2}) + 4(1 + 2y^2) + 160 \right] \beta^2$$

$$+ \left[\frac{52}{25} (10 - 1 - \frac{26}{25} y^2 + y) \right] - (4y^2 - 6y + \frac{8}{5} y + 2(1 + 2y^2 + 40)) (\frac{102}{25} y - 1) \beta \left[\beta \right.$$

$$\left. + y(2y - \frac{3}{2})(10 - 1) - \frac{26}{25} y^2 + y \right] = 0$$

$$\frac{52}{25}$$

$$\left[\frac{3848}{625} \gamma \right] = A_3$$

$$\left[\frac{244}{25} \gamma^2 - \frac{33}{25} \gamma + 16\theta + 4 \right] = A_2$$

$$\left[\frac{6348}{625} \gamma^3 + \frac{102}{25} \gamma^2 + \left(\frac{236}{5} \theta + \frac{383}{100} \right) \gamma - 2(1+4\theta) \right] = A_1$$

$$\gamma \left[-\frac{52}{25} \gamma^3 + \frac{89}{25} \gamma^2 + \left(14\theta - \frac{7}{2} \right) \gamma - \frac{3}{2}(7\theta - 1) \right] = A_0$$

$$A_3 \beta^3 + A_2 \beta^2 + A_1 \beta + A_0 = 0$$

$$\text{Let } \theta = 0.001 \quad \underline{\underline{\gamma = 1.}}$$

$$A_3 = 6.1568$$

$$A_2 = 9.76 - 1.32 + 0.016 + 4 = 12.456$$

$$A_1 = 10.1568 + 4.08 + 0.0472 + 3.83 - 2.008 = 16.1060$$

$$A_0 = -2.08 + 3.56 + 0.014 - 3.5 + 1.4895 = -0.5165$$

$$f(\beta) = \beta^3 + 2.02313 \beta^2 + 2.61597 \beta - 0.0838910 = 0$$

$$f'(\beta) = 3\beta^2 + 4.04626 \beta + 2.61597$$

$$f(0.031) = -0.0008218$$

$$f'(0.031) = 2.74429$$

$$f(0.0312995) = 0$$

$$\underline{\underline{\beta = 0.0312995}}$$

$$\beta^2 + 2.05443 \beta + 2.68027 = 0$$

$$\beta = -1.02722 \pm \sqrt{1.02722^2 - 2.68027} \quad \text{Complex.}$$

001

$$\frac{\beta \left[(1+2\gamma^2) + 4\theta \right] \left[\frac{52}{25} \gamma - 1 + 2\beta \right]}{\gamma \left[\frac{52}{25} \beta - \frac{3}{2} + 2\gamma \right]} + (2\beta^2 + 4\theta) = 4\lambda$$

$$\lambda = \frac{1}{4} \left[\frac{0.0312995 \times 3.004 \times 1.14260}{0.5312995} + 0.0059593 \right] = 0.052041$$

$$\frac{\sigma}{E} m^2 = 0.052041$$

$$\text{Let } m = 12$$

$$\frac{1}{3(1-\nu^2)} \left(\frac{\sigma}{E} m^2 \right)^2 = 0.001$$

$$\left(\frac{R}{E} \right)^2 = \frac{144^2 \cdot 10^3}{3(1-\nu^2)}$$

$$\frac{\sigma}{E} = 0.0003614$$

$$\left(\frac{R}{E} \right) = \frac{144 \times 31.62278}{1.65227}$$

$$\frac{\sigma}{E} \frac{R}{E} = \underline{0.996} \quad !!!$$

$$= 2756$$

$$\alpha^2 = 8 \left[0.052041 \times - \frac{24}{25} (1.00271) + 0.078062 \times 1 - 1.04082 - 0.001 \right]$$

$$= (-)$$

Impossible !!!

$$\gamma = -1$$

602

$$A_3 = -6.1568$$

$$A_2 = 9.76 + 1.32 + 0.016 + 4 = 15.096$$

$$A_1 = -10.1568 + 4.08 - 0.0472 - 3.83 - 2.008 = -11.962$$

$$A_0 = -2.08 - 3.56 + 0.014 - 3.5 - 1.4895 = -10.6155$$

$$f(\beta) = \beta^3 - 2.45192\beta^2 + 1.94289\beta + 1.72419 = 0$$

$$f'(\beta) = 3\beta^2 - 4.90384\beta + 1.94289$$

$$\left. \begin{aligned} f(\beta) &= \beta^3 - 2.45192\beta^2 + 1.94289\beta + 1.72419 \\ f'(\beta) &= 3\beta^2 - 4.90384\beta + 1.94289 \end{aligned} \right\}$$

$$f(0.50) = -0.01476$$

$$f'(0.50) = 5.1448$$

$$f(0.50287) = +0.00004$$

$$f'(0.50287) = 5.16$$

$$\underline{\underline{\beta = -0.50286}}$$

$$\beta^2 - 2.95478\beta + 3.42873 = 0$$

$$\beta = 1.47738 \pm \sqrt{1.47738^2 - 3.42873} \quad \text{Complex}$$

$$m^2 f_1 = \delta$$

$$\boxed{\delta = \frac{1}{8} \alpha^2 + \gamma^2 - 4\lambda}$$

$$\left| \lambda \frac{1}{4} \left[\frac{0.50286 \times 3.004 \times (-2.02425)}{-5.58000} + 0.25682 \right] \right|$$

= too Big !!!

lot of mistakes!!!

if $\alpha=0$

$$0 = 1 + 2\beta^2 + 4\theta$$

$$2\lambda = \beta^2 + 2\theta$$

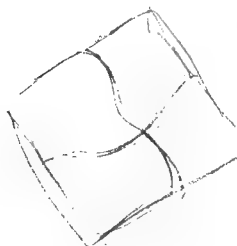
$$\lambda = 0$$

thus

$$\frac{\sigma}{E} m^2 = \frac{1}{3(1-\nu^2)} \left(\frac{t}{R} m^2 \right)^2$$

$$\frac{\sigma}{E} \frac{R}{t} = \frac{1}{3(1-\nu^2)} \left(\frac{t}{R} m^2 \right)$$

$$\frac{6.1}{1.4} = 5.2$$



(5.0)

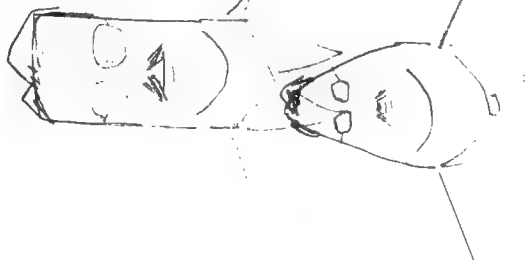
$$\frac{3(1-\nu^2)}{\left(\frac{t}{R}\right)^2} \frac{\sigma}{E} = m^2$$



$$1.5 = \frac{1}{4} + \frac{1}{2} (1)$$

$$\frac{9}{4} = \frac{1}{4}$$

$$\frac{9}{64} \frac{9}{53}$$



$$\begin{aligned}
\frac{w}{R} &= f_0 + f_1 \cos^2 \frac{m(x+y)}{2R} \cos^2 \frac{m(x-y)}{2R} \\
&= f_0 + f_1 \left\{ \cos^2 \frac{mx}{2R} \cos^2 \frac{my}{2R} - \sin^2 \frac{mx}{2R} \sin^2 \frac{my}{2R} \right\}^2 \\
&= f_0 + f_1 \left\{ 1 - \sin^2 \frac{mx}{2R} - \sin^2 \frac{my}{2R} \right\}^2 \\
&= f_0 + \frac{1}{4} f_1 \left\{ \cos \frac{mx}{R} + \cos \frac{my}{R} \right\}^2 \\
&= f_0 + \frac{1}{4} f_1 \left\{ \cos^2 \frac{mx}{R} + 2 \cos \frac{mx}{R} \cos \frac{my}{R} + \cos^2 \frac{my}{R} \right\} \\
&= f_0 + \frac{1}{4} f_1 \left\{ \frac{1}{2} (1 + \cos \frac{2mx}{R}) + 2 \cos \frac{mx}{R} \cos \frac{my}{R} + \frac{1}{2} (1 + \cos \frac{2my}{R}) \right\} \\
&= (f_0 + \frac{1}{4} f_1) + \frac{1}{8} f_1 \cos \frac{2mx}{R} + \frac{1}{8} f_1 \cos \frac{2my}{R} + \frac{1}{2} f_1 \cos \frac{mx}{R} \cos \frac{my}{R} \\
&= (f_0 + \frac{1}{4} f_1) + \frac{1}{2} f_1 \left[\cos \frac{mx}{R} \cos \frac{my}{R} + \frac{1}{4} \cos \frac{2mx}{R} + \frac{1}{4} \cos \frac{2my}{R} \right]
\end{aligned}$$

$$\frac{1}{R} \frac{\partial^2 w}{\partial x^2} = -\left(\frac{m}{R}\right)^2 \frac{1}{2} f_1 \left[\cos \frac{mx}{R} \cos \frac{my}{R} + \cos \frac{2mx}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 w}{\partial y^2} = -\left(\frac{m}{R}\right)^2 \frac{1}{2} f_1 \left[\cos \frac{mx}{R} \cos \frac{my}{R} + \cos \frac{2my}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 w}{\partial x \partial y} = \left(\frac{m}{R}\right)^2 \frac{1}{2} f_1 \left[\sin \frac{mx}{R} \sin \frac{my}{R} \right]$$

$$\begin{aligned}
 \Delta \Delta F &= E \frac{m^2}{R^2} \frac{f_1}{2} \left[\left(\frac{f_1}{2} m^2 \right) \left\{ -\frac{1}{2} \cos \frac{2mX}{R} - \frac{1}{2} \cos \frac{2mY}{R} \right. \right. \\
 &\quad - \frac{1}{2} \left(\cos \frac{3mX}{R} + \cos \frac{mX}{R} \right) \cos \frac{mY}{R} - \frac{1}{2} \cos \frac{mX}{R} \left(\cos \frac{3mY}{R} + \cos \frac{mY}{R} \right) \\
 &\quad \left. \left. - \cos \frac{2mX}{R} \cos \frac{2mY}{R} \right\} + \cos \frac{mX}{R} \cos \frac{mY}{R} + \cos \frac{2mX}{R} \right] \\
 &= E \left(\frac{m}{R} \right)^2 \frac{f_1}{2} \left[\left(1 - \frac{f_1 m^2}{2} \right) \cos \frac{mX}{R} \cos \frac{mY}{R} + \left(1 - \frac{f_1 m^2}{4} \right) \cos \frac{2mX}{R} \right. \\
 &\quad - \frac{f_1 m^2}{4} \cos \frac{2mY}{R} - \frac{f_1 m^2}{2} \cos \frac{2mX}{R} \cos \frac{2mY}{R} - \frac{f_1 m^2}{4} \cos \frac{3mX}{R} \cos \frac{mY}{R} \\
 &\quad \left. - \frac{f_1 m^2}{4} \cos \frac{mX}{R} \cos \frac{3mY}{R} \right]
 \end{aligned}$$

$$\begin{aligned}
 F &= E \left(\frac{R}{m} \right)^2 \frac{f_1}{2} \left[\frac{1}{4} \left(1 - \frac{f_1 m^2}{2} \right) \cos \frac{mX}{R} \cos \frac{mY}{R} + \frac{1}{16} \left(1 - \frac{f_1 m^2}{4} \right) \cos \frac{2mX}{R} \right. \\
 &\quad - \frac{f_1 m^2}{64} \cos \frac{2mY}{R} - \frac{f_1 m^2}{128} \cos \frac{2mX}{R} \cos \frac{2mY}{R} - \frac{f_1 m^2}{400} \cos \frac{3mX}{R} \cos \frac{mY}{R} \\
 &\quad \left. - \frac{f_1 m^2}{400} \cos \frac{mX}{R} \cos \frac{3mY}{R} \right] - \frac{\tilde{\sigma}}{2} \eta^2 + \frac{1}{2} \chi^2
 \end{aligned}$$

$$\begin{aligned}
 \tilde{\sigma}_x &= E \frac{f_1}{2} \left[\frac{1}{4} \left(\frac{f_1 m^2}{2} - 1 \right) \cos \frac{mX}{R} \cos \frac{mY}{R} + \frac{f_1 m^2}{16} \cos \frac{2mX}{R} + \frac{f_1 m^2}{32} \cos \frac{2mY}{R} \cos \frac{2mX}{R} \right. \\
 &\quad \left. + \frac{f_1 m^2}{400} \cos \frac{3mX}{R} \cos \frac{mY}{R} + \frac{9 f_1 m^2}{400} \cos \frac{mX}{R} \cos \frac{3mY}{R} \right] - \sigma
 \end{aligned}$$

$$\sigma_y = E \frac{f_1}{2} \left[\frac{1}{4} \left(\frac{f_1 m^2}{2} - 1 \right) \cos \frac{mx}{R} \cos \frac{ny}{R} + \frac{1}{4} \left(\frac{f_1 m^2}{4} - 1 \right) \cos \frac{2mx}{R} \right. \\ \left. + \frac{f_1 m^2}{32} \cos \frac{2mx}{R} \cos \frac{2ny}{R} + \frac{9 f_1 m^2}{400} \cos \frac{3mx}{R} \cos \frac{ny}{R} + \frac{f_1 m^2}{400} \cos \frac{mx}{R} \cos \frac{3ny}{R} \right] + 1 \quad \underline{606}$$

$$\tau_{xy} = E \frac{f_1}{2} \left[\frac{1}{4} \left(\frac{f_1 m^2}{2} - 1 \right) \sin \frac{mx}{R} \sin \frac{ny}{R} + \frac{f_1 m^2}{32} \sin \frac{2mx}{R} \sin \frac{2ny}{R} \right. \\ \left. + \frac{3}{400} f_1 m^2 \sin \frac{3mx}{R} \sin \frac{ny}{R} + \frac{3 f_1 m^2}{400} \sin \frac{mx}{R} \sin \frac{3ny}{R} \right]$$

$$\frac{1}{E} (\sigma_y - \nu \sigma_x) = \frac{1}{E} (\lambda + \nu \sigma) + \frac{f_1}{2} \left[\frac{1-\nu}{4} \left(\frac{f_1 m^2}{2} - 1 \right) \cos \frac{mx}{R} \cos \frac{ny}{R} + \frac{1}{4} \left(\frac{f_1 m^2}{4} - 1 \right) \cos \frac{2mx}{R} \right. \\ \left. - \nu \frac{f_1 m^2}{16} \cos \frac{2mx}{R} + \frac{1-\nu}{32} f_1 m^2 \cos \frac{2mx}{R} \cos \frac{2ny}{R} + \frac{9-\nu}{400} f_1 m^2 \cos \frac{3mx}{R} \cos \frac{ny}{R} \right. \\ \left. + \frac{1-9\nu}{400} f_1 m^2 \cos \frac{mx}{R} \cos \frac{3ny}{R} \right]$$

$$-\frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 = \frac{f_1}{2} \left[-\frac{f_1 m^2}{4} \left(\cos \frac{mx}{R} \sin \frac{ny}{R} + \frac{1}{2} \sin \frac{2mx}{R} \right)^2 \right] \\ = \frac{f_1}{2} \left[-\frac{f_1 m^2}{4} \left\{ \cos^2 \frac{mx}{R} \sin^2 \frac{ny}{R} + \frac{1}{2} \cos \frac{mx}{R} \left(\cos \frac{ny}{R} - \cos \frac{3ny}{R} \right) \right. \right. \\ \left. \left. + \frac{1}{8} (1 - \cos \frac{4ny}{R}) \right\} \right]$$

$$= \frac{f_1}{2} \left[-\frac{f_1 m^2}{4} \left\{ \frac{1}{4} (1 + \cos \frac{2mx}{R}) (1 - \cos \frac{2ny}{R}) + \frac{1}{2} \cos \frac{mx}{R} \cos \frac{ny}{R} \right. \right. \\ \left. \left. - \frac{1}{2} \cos \frac{3mx}{R} \cos \frac{3ny}{R} + \frac{1}{8} - \frac{1}{8} \cos \frac{4ny}{R} \right\} \right]$$

$$= \frac{f_1}{2} \left[-\frac{f_1 m^2}{4} \left\{ \frac{3}{8} + \frac{1}{4} \cos \frac{2mx}{R} - \frac{1}{4} \cos \frac{2ny}{R} - \frac{1}{4} \cos \frac{2mx}{R} \cos \frac{2ny}{R} \right. \right. \\ \left. \left. + \frac{1}{2} \cos \frac{mx}{R} \cos \frac{ny}{R} - \frac{1}{2} \cos \frac{3mx}{R} \cos \frac{3ny}{R} - \frac{1}{8} \cos \frac{4ny}{R} \right\} \right]$$

$$\frac{1}{4}f_1 + \frac{1}{E}(\lambda + v\sigma) + f_0 - \frac{3}{64}f_1(f_1 m^2) = 0$$

602

$$\boxed{\frac{\lambda}{E} = \frac{3}{64}f_1(f_1 m^2) - (f_0 + \frac{f_1}{4}) - v\frac{\sigma}{E}}$$

The increase in potential energy

$$-\left[+\frac{1}{E}(\sigma + v\lambda) + \frac{3}{64}f_1(f_1 m^2) \right] 8 \frac{\sigma}{E}$$

$$\boxed{f_1 = -8 \frac{\sigma}{E} \left[(1-v^2) \frac{\sigma}{E} + (1+v) \frac{3}{64}f_1(f_1 m^2) - v(f_0 + \frac{f_1}{4}) \right]}$$

$$\begin{aligned} \sigma_x + \sigma_y &= (-\sigma + \lambda) + E \frac{f_1}{2} \left[\frac{1}{2} \left(\frac{f_1 m^2}{2} - 1 \right) \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + \frac{1}{4} \left(\frac{f_1 m^2}{4} - 1 \right) \cos \frac{2\pi x}{R} \right. \\ &+ \frac{f_1 m^2}{16} \cos \frac{2\pi y}{R} + \frac{f_1 m^2}{16} \cos \frac{2\pi x}{R} \cos \frac{3\pi x}{R} + \frac{1}{40} f_1 m^2 \cos \frac{3\pi y}{R} \cos \frac{\pi y}{R} \\ &\left. + \frac{1}{40} f_1 m^2 \cos \frac{\pi x}{R} \cos \frac{3\pi x}{R} \right] \end{aligned}$$

$$\begin{aligned} f_2 &= 4 \left(\frac{\sigma - \lambda}{E} \right)^2 + \frac{f_1^2}{4} \left[\frac{1}{4} \left(\frac{f_1 m^2}{2} - 1 \right)^2 + \frac{1}{8} \left(\frac{f_1 m^2}{4} - 1 \right)^2 + \frac{1}{128} (f_1 m^2)^2 \right. \\ &\left. + \frac{(f_1 m^2)^2}{256} + \frac{2(f_1 m^2)^2}{1600} \right] \end{aligned}$$

$$\begin{aligned} &= 4 \left(\frac{\sigma - \lambda}{E} \right)^2 + \frac{f_1^2}{16} \left[\frac{(f_1 m^2)^2}{4} - (f_1 m^2) + 1 + \frac{(f_1 m^2)^2}{32} - \frac{1}{4} (f_1 m^2) + \frac{1}{2} \right. \\ &\left. + \left(\frac{1}{32} + \frac{1}{64} + \frac{1}{200} \right) (f_1 m^2)^2 \right] \end{aligned}$$

$$f_2 = 4 \left(\frac{\sigma - \lambda}{E} \right)^2 + \frac{f_1^2}{16} \left[\left(\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{200} \right) (f_1 m^2)^2 - \frac{5}{4} (f_1 m^2) + \frac{3}{2} \right] \quad \underline{\underline{608}}$$

$$= 4 \left(\frac{\sigma - \lambda}{E} \right)^2 + \frac{f_1^2}{16} \left[\frac{533}{1600} (f_1 m^2)^2 - \frac{5}{4} (f_1 m^2) + \frac{3}{2} \right] + 8(1+\nu) \frac{\sigma \lambda}{E}$$

$$\boxed{f_2 = 4 \left(\frac{\sigma - \lambda}{E} \right)^2 + \frac{f_1^2}{64} \left[\frac{533}{400} (f_1 m^2)^2 - 5 (f_1 m^2) + 6 \right] + 8(1+\nu) \frac{\sigma \lambda}{E}}$$

$$f_3 = \frac{1}{12(1-\nu^2)} \left(\frac{t}{R} \right)^2 m^4 \frac{f_1^2}{4} [4 + 2 + 2]$$

$$\boxed{f_3 = \frac{1}{6(1-\nu^2)} \left(\frac{t}{R} \right)^2 m^4 f_1^2}$$

$$4 \left(\frac{\sigma - \lambda}{E} \right)^2 = 4 \left\{ (1+\nu) \frac{\sigma}{E} - \frac{3}{64} f_1 (f_1 m^2) + \left(f_0 + \frac{f_1}{4} \right) \right\}^2$$

$$= 4 \left\{ (1+\nu)^2 \left(\frac{\sigma}{E} \right)^2 - \frac{3}{32} (1+\nu) \frac{\sigma}{E} f_1 (f_1 m^2) + 2(1+\nu) \left(f_0 + \frac{f_1}{4} \right) \frac{\sigma}{E} \right.$$

$$\left. + \frac{9}{64} f_1^2 (f_1 m^2)^2 + \left(f_0 + \frac{f_1}{4} \right)^2 - \frac{3}{32} f_1 (f_1 m^2) \left(f_0 + \frac{f_1}{4} \right) \right\}$$

$$\cancel{f_1 + 4 \left(\frac{\sigma - \lambda}{E} \right)^2 = 4 \left(\frac{\sigma}{E} \right)^2 \left\{ (1+\nu)^2 - 2(1-\nu^2) \right\} - \frac{3}{4} (1+\nu) \frac{\sigma}{E} f_1 (f_1 m^2)} \\ + \frac{\sigma}{E} \left\{ 8(1+\nu) \left(f_0 + \frac{f_1}{4} \right) + \frac{9}{1024} f_1^2 (f_1 m^2)^2 + 4 \left(f_0 + \frac{f_1}{4} \right)^2 - \frac{3}{8} f_1 (f_1 m^2) \left(f_0 + \frac{f_1}{4} \right) \right\}$$

$$\frac{64}{384}$$

$$1 + \nu - 2 + 2 - \nu - \nu$$

$$8(1+v)\frac{\sigma_1}{E} = 8(1+v)\frac{\sigma}{E} \left[\frac{3}{64} f_1(f_1 m^2) - (f_0 + \frac{f_1}{4}) - v\frac{\sigma}{E} \right] \quad \underline{609}$$

$$8f_1 + 4\left(\frac{\sigma_1}{E}\right)^2 + 2(1+v)4\frac{\sigma_1}{E} = K$$

$$= 4\left(\frac{\sigma}{E}\right)^2 \left\{ (1+v)^2 - 2(1-v) - 2v(1+v) \right\} - \frac{3}{8}(1+v)\frac{\sigma}{E} f_1(f_1 m^2) + 8v\frac{\sigma}{E} \left(f_0 + \frac{f_1}{4}\right) \\ + \frac{9}{1024} f_1^2(f_1 m^2)^2 + 4\left(f_0 + \frac{f_1}{4}\right)^2 - \frac{3}{8} f_1(f_1 m^2) \left(f_0 + \frac{f_1}{4}\right)$$

$$\frac{\partial K}{\partial f_0} = 0 \quad \text{gives} \quad \boxed{v\frac{\sigma}{E} + \left(f_0 + \frac{f_1}{4}\right) - \frac{3}{64} f_1(f_1 m^2) = 0}$$

$$\therefore K = -4\left(\frac{\sigma}{E}\right)^2(1-v) - \frac{3}{8}(1+v)\frac{\sigma}{E} f_1(f_1 m^2) + \frac{9}{1024} f_1^2(f_1 m^2)^2 - 4\left(f_0 + \frac{f_1}{4}\right)^2$$

$$\text{But} \quad 4\left(f_0 + \frac{f_1}{4}\right)^2 = \left\{ 2v\frac{\sigma}{E} - \frac{3}{32} f_1(f_1 m^2) \right\}^2$$

$$= 4\left(\frac{\sigma}{E}\right)^2 v^2 - \frac{3}{8} v\frac{\sigma}{E} f_1(f_1 m^2) + \frac{9}{1024} f_1^2(f_1 m^2)^2$$

$$\begin{array}{r} 32 \\ 72 \\ 64 \\ \hline 40 \\ 1024 \end{array}$$

$$\therefore K = -4\left(\frac{\sigma}{E}\right)^2 - \frac{3}{8}\left(\frac{\sigma}{E}\right) f_1(f_1 m^2)$$

The potential of the system.

$$P = -4\left(\frac{\sigma}{E}\right)^2 - \frac{3}{8}\left(\frac{\sigma}{E}\right) f_1(f_1 m^2) + \frac{f_1^2}{64} \left[\frac{533}{400} (f_1 m^2)^2 - 5(f_1 m^2) + 6 \right] + \frac{1}{6(1-v)} \left(\frac{f_1}{R}\right)^2 m^4$$

$$\frac{\partial P}{\partial f_1} = 0$$

$$\frac{3}{4}\left(\frac{\sigma}{E}\right) m^2 = \frac{1}{32} \left[\frac{533}{200} (f_1 m^2)^2 - 7.5(f_1 m^2) + 6 \right] + \frac{1}{3(1-v)} \left(\frac{f_1}{R}\right)^2 m^4$$

6/0

$$\frac{\sigma}{E} = \left[\frac{533}{4800} f_1^2 m^2 - \frac{15}{48} f_1 + \frac{1}{4} \frac{1}{m^2} \right] + \frac{4}{9(1-\nu^2)} \left(\frac{f}{R} \right)^2 m^2$$

$$= \left\{ \frac{533}{4800} f_1^2 + \frac{4}{9(1-\nu^2)} \left(\frac{f}{R} \right)^2 \right\} m^2 + \frac{1}{4} \frac{1}{m^2} - \frac{5}{16} f_1$$

$$= 2 \left\{ \frac{533}{12 \times (40)^2} f_1^2 + \frac{1}{9(1-\nu^2)} \left(\frac{f}{R} \right)^2 \right\}^{\frac{1}{2}} - \frac{5}{16} f_1$$

$$\boxed{\frac{\sigma}{E} \frac{R}{t} = \left\{ \frac{533}{3 \times (40)^2} \left(\frac{f}{t} \right)^2 + \frac{4}{9(1-\nu^2)} \right\}^{\frac{1}{2}} - \frac{5}{16} \left(\frac{f}{t} \right)}$$

$$\frac{\partial \left(\frac{\sigma}{E} \frac{R}{t} \right)}{\partial \left(\frac{f}{t} \right)} = 0,$$

$$\frac{\frac{1}{2} \times 2 \frac{533}{3 \times 1600} \left(\frac{f}{t} \right)}{\left\{ \frac{533}{3 \times 1600} \left(\frac{f}{t} \right)^2 + \frac{4}{9(1-\nu^2)} \right\}^{\frac{1}{2}}} = \frac{5}{16}$$

$$\left(\frac{533}{4800} \right) \left(\frac{f}{t} \right)^2 = \frac{25}{256} \left\{ \frac{533}{4800} \left(\frac{f}{t} \right)^2 + \frac{4}{9(1-\nu^2)} \right\}$$

$$\frac{533}{4800} \left(\frac{533}{4800} - \frac{25}{256} \right) \left(\frac{f}{t} \right)^2 = \frac{25}{9 \times 64 (1-\nu^2)}$$

$$\frac{533}{75} \times \frac{1}{64} \left(\frac{533}{75} - \frac{25}{4} \right) \left(\frac{f}{t} \right)^2 = \frac{25}{9(1-\nu^2)}$$

$$\frac{533}{75} \times \frac{1}{64} \times \frac{1}{75} \times \frac{1}{4} (257) \left(\frac{f}{t} \right)^2 = \frac{25}{9(1-\nu^2)}$$

$$\left(\frac{\delta}{t}\right)^2 = \frac{1}{1-\nu^2} \frac{62500 \times 64}{533 \times 257} = \frac{4000000}{124652.71}$$

$$\left(\frac{\delta}{t}\right)^2 = \frac{1}{1-\nu^2} \frac{62500 \times 64}{533 \times 257} = \frac{4000000}{124652.71}$$

$$\left(\frac{f}{E}\right)^2 = 32.0892 \quad \frac{f}{E} = 5.6648$$

$$\left(\frac{\sigma}{E} \frac{R}{t}\right)_{\min} = 5.6648 \left\{ \frac{533}{\frac{460}{300}} \frac{7}{5} - \frac{5}{16} \right\}$$

$$= 5.6648 \left\{ \frac{5.33}{15} - \frac{5}{16} \right\} = 5.6648 \{ 0.355333 - 0.312500 \}$$

$$= \underline{\underline{0.24264}} \quad \dots$$

$$\frac{0.24264}{0.606} = \underline{0.400}$$

$$\left(\frac{f}{T}\right) = 18.02912$$

$$\frac{FR}{Et} = \left\{ \frac{533}{4800} (18.02912)^2 + \frac{4}{8.19} \right\}^{\frac{1}{2}} - \frac{5}{16} \times 18.02912$$

$$= (36.09401 + 0.48840)^{\frac{1}{2}} = 5.6341$$

$$= \underline{\underline{0.41426}}$$

00-00-00-00-00-00

$$\begin{aligned}
\frac{W}{R} &= f_0 + f_1 \left[\cos^4 \frac{m(x+y)}{2R} \cos^2 \frac{n(x-y)}{2R} \right] \\
&= f_0 + f_1 \left[\frac{1}{16} \left\{ \cos \frac{nx}{R} + \cos \frac{ny}{R} \right\}^4 \right] \\
&= f_0 + f_1 \left[\frac{1}{16} \left\{ \cos^2 \frac{nx}{R} + 2 \cos \frac{nx}{R} \cos \frac{ny}{R} + \cos^2 \frac{ny}{R} \right\}^2 \right] \\
&= f_0 + f_1 \left[\frac{1}{16} \left\{ 1 + \frac{1}{2} \cos \frac{2nx}{R} + \frac{1}{2} \cos \frac{2ny}{R} + 2 \cos \frac{nx}{R} \cos \frac{ny}{R} \right\}^2 \right] \\
&= f_0 + f_1 \left[\frac{1}{16} \left\{ 1 + \frac{1}{4} \cos^2 \frac{2nx}{R} + \frac{1}{4} \cos^2 \frac{2ny}{R} + \frac{1}{2} \cos \frac{2nx}{R} \cos \frac{2ny}{R} \right. \right. \\
&\quad \left. \left. + \cos \frac{2nx}{R} + \cos \frac{2ny}{R} + 4 \cos \frac{nx}{R} \cos \frac{ny}{R} + \frac{1}{2} \cos \frac{2nx}{R} \cos \frac{2ny}{R} \right. \right. \\
&\quad \left. \left. + (\cos \frac{3nx}{R} + \cos \frac{nx}{R}) \cos \frac{ny}{R} + \cos \frac{ny}{R} (\cos \frac{3ny}{R} + \cos \frac{ny}{R}) \right\} \right] \\
&= f_0 + f_1 \left[\frac{1}{16} \left\{ 1 + \frac{1}{8} + \frac{1}{8} \cos \frac{4nx}{R} + \frac{1}{8} + \frac{1}{8} \cos \frac{4ny}{R} \right. \right. \\
&\quad \left. \left. + 1 + \cos \frac{2nx}{R} + \cos \frac{2ny}{R} + \cos \frac{2nx}{R} \cos \frac{2ny}{R} + \cos \frac{2nx}{R} + \cos \frac{2ny}{R} \right. \right. \\
&\quad \left. \left. + 6 \cos \frac{nx}{R} \cos \frac{ny}{R} + \frac{1}{2} \cos \frac{2nx}{R} \cos \frac{2ny}{R} + \cos \frac{3nx}{R} \cos \frac{ny}{R} + \cos \frac{ny}{R} \cos \frac{3ny}{R} \right\} \right] \\
\hline
\frac{W}{R} &= f_0 + f_1 \left[\frac{9}{64} + \frac{3}{8} \cos \frac{nx}{R} \cos \frac{ny}{R} + \frac{1}{8} \cos \frac{2nx}{R} + \frac{1}{8} \cos \frac{2ny}{R} \right. \\
&\quad \left. + \frac{3}{32} \cos \frac{2nx}{R} \cos \frac{2ny}{R} + \frac{1}{16} \cos \frac{3nx}{R} \cos \frac{ny}{R} + \frac{1}{16} \cos \frac{nx}{R} \cos \frac{3ny}{R} \right. \\
&\quad \left. + \frac{1}{128} \cos \frac{4nx}{R} + \frac{1}{128} \cos \frac{4ny}{R} \right]
\end{aligned}$$

$$\frac{\psi}{R} = \left(f_0 + \frac{9}{64} f_1\right) + \frac{f_1}{8} \left[3 \cos \frac{mx}{R} \cos \frac{ny}{R} + \cos \frac{2mx}{R} + \cos \frac{2ny}{R} \right. \\ \left. + \frac{3}{4} \cos \frac{2mx}{R} \cos \frac{2ny}{R} + \frac{1}{2} \cos \frac{3mx}{R} \cos \frac{ny}{R} + \frac{1}{2} \cos \frac{mx}{R} \cos \frac{3ny}{R} \right. \\ \left. + \frac{1}{16} \cos \frac{4mx}{R} + \frac{1}{16} \cos \frac{4ny}{R} \right] \quad \underline{\underline{613}}$$

$$\left(\frac{\partial \psi}{\partial y}\right) = m \frac{f_1}{8} \left[3 \cos \frac{mx}{R} \sin \frac{ny}{R} + 2 \sin \frac{2ny}{R} + \frac{3}{2} \cos \frac{2mx}{R} \sin \frac{ny}{R} \right. \\ \left. + \frac{1}{2} \cos \frac{3mx}{R} \sin \frac{ny}{R} + \frac{3}{2} \cos \frac{mx}{R} \sin \frac{3ny}{R} + \frac{1}{4} \sin \frac{4ny}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 \psi}{\partial x^2} = -\left(\frac{m}{R}\right)^2 \frac{f_1}{8} \left[3 \cos \frac{mx}{R} \cos \frac{ny}{R} + 4 \cos \frac{2mx}{R} + 3 \cos \frac{2mx}{R} \cos \frac{2ny}{R} \right. \\ \left. + \frac{9}{2} \cos \frac{3mx}{R} \cos \frac{ny}{R} + \frac{1}{2} \cos \frac{mx}{R} \cos \frac{3ny}{R} + \cos \frac{4mx}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 \psi}{\partial y^2} = -\left(\frac{m}{R}\right)^2 \frac{f_1}{8} \left[3 \cos \frac{mx}{R} \cos \frac{ny}{R} + 4 \cos \frac{2ny}{R} + 3 \cos \frac{2mx}{R} \cos \frac{2ny}{R} \right. \\ \left. + \frac{1}{2} \cos \frac{3mx}{R} \cos \frac{ny}{R} + \frac{9}{2} \cos \frac{mx}{R} \cos \frac{3ny}{R} + \cos \frac{4ny}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 \psi}{\partial x \partial y} = \left(\frac{m}{R}\right)^2 \frac{f_1}{8} \left[3 \sin \frac{mx}{R} \sin \frac{ny}{R} + 3 \sin \frac{2mx}{R} \sin \frac{2ny}{R} + \frac{3}{2} \sin \frac{3mx}{R} \sin \frac{ny}{R} \right. \\ \left. + \frac{3}{2} \sin \frac{mx}{R} \sin \frac{3ny}{R} \right]$$

$$\frac{3}{2} + \frac{27}{2} :$$

$$\begin{aligned}
\Delta F = & \left(\frac{m}{R}\right)^2 E \frac{f}{8} \left[\left(\frac{f}{8} m^2\right) \right] \left\{ 9 \left(\sin^2 \frac{mX}{R} \sin^2 \frac{mY}{R} - \cos^2 \frac{mX}{R} \cos^2 \frac{mY}{R} \right) \right. \\
& + 9 \left(\sin^2 \frac{2mX}{R} \sin^2 \frac{2mY}{R} - \cos^2 \frac{2mX}{R} \cos^2 \frac{2mY}{R} \right) \\
& + \frac{9}{4} \left(\sin^2 \frac{3mX}{R} \sin^2 \frac{mY}{R} - \cos^2 \frac{3mX}{R} \cos^2 \frac{mY}{R} \right) \\
& + \frac{9}{4} \left(\sin^2 \frac{mX}{R} \sin^2 \frac{3mY}{R} - \cos^2 \frac{mX}{R} \cos^2 \frac{3mY}{R} \right) \\
& + 18 \left(\sin \frac{mX}{R} \sin \frac{2mY}{R} \sin \frac{mY}{R} \sin \frac{2mX}{R} - \cos \frac{mX}{R} \cos \frac{2mY}{R} \cos \frac{mY}{R} \cos \frac{2mX}{R} \right) \quad (1) \\
& + 3 \left(3 \sin \frac{mX}{R} \sin \frac{3mX}{R} \sin^2 \frac{mY}{R} - 5 \cos \frac{mX}{R} \cos \frac{3mX}{R} \cos^2 \frac{mY}{R} \right) \\
& + 3 \left(3 \sin^2 \frac{mY}{R} \sin \frac{mX}{R} \sin \frac{3mX}{R} - 5 \cos^2 \frac{mY}{R} \cos \frac{mX}{R} \cos \frac{3mX}{R} \right) \\
& + 3 \left(3 \sin \frac{2mY}{R} \sin \frac{3mX}{R} \sin \frac{mX}{R} \sin \frac{2mY}{R} - 5 \cos \frac{2mY}{R} \cos \frac{3mX}{R} \cos \frac{mX}{R} \cos \frac{2mY}{R} \right) \\
& + 3 \left(3 \sin \frac{mY}{R} \sin \frac{2mX}{R} \sin \frac{2mY}{R} \sin \frac{3mX}{R} - 5 \cos \frac{mY}{R} \cos \frac{2mX}{R} \cos \frac{2mY}{R} \cos \frac{3mX}{R} \right) \\
& + 9 \left(\sin \frac{mY}{R} \sin \frac{3mY}{R} \sin \frac{mX}{R} \sin \frac{3mX}{R} - \cos \frac{mY}{R} \cos \frac{3mY}{R} \cos \frac{mX}{R} \cos \frac{3mX}{R} \right) \quad (1) \\
& - 12 \cos \frac{mX}{R} \cos \frac{mY}{R} \cos \frac{2mY}{R} - 16 \cos \frac{2mX}{R} \cos \frac{2mY}{R} - 12 \cos \frac{2mX}{R} \cos^2 \frac{2mY}{R} \\
& - 18 \cos \frac{3mX}{R} \cos \frac{mY}{R} \cos \frac{2mY}{R} - 2 \cos \frac{mX}{R} \cos \frac{2mY}{R} \cos \frac{3mY}{R} - 4 \cos \frac{4mX}{R} \cos \frac{2mY}{R} \\
& - 3 \cos \frac{mX}{R} \cos \frac{mY}{R} \cos \frac{4mY}{R} - 4 \cos \frac{2mX}{R} \cos \frac{4mY}{R} - 3 \cos \frac{2mX}{R} \cos \frac{2mY}{R} \cos \frac{4mY}{R} \\
& - \frac{9}{2} \cos \frac{3mX}{R} \cos \frac{mY}{R} \cos \frac{4mY}{R} - \frac{1}{2} \cos \frac{mX}{R} \cos \frac{3mY}{R} \cos \frac{4mY}{R} - \cos \frac{4mX}{R} \cos \frac{4mY}{R}
\end{aligned}$$

$$\begin{aligned}
& + 12 \cos \frac{mx}{R} \cos \frac{2mx}{R} \cos \frac{mx}{R} - 16 \cos \frac{2mx}{R} \cos \frac{2mx}{R} - 12 \cos \frac{2mx}{R} \cos \frac{2mx}{R} \quad \underline{\underline{6/5}} \\
& + 2 \cos \frac{2mx}{R} \cos \frac{3mx}{R} \cos \frac{mx}{R} - 18 \cos \frac{mx}{R} \cos \frac{2mx}{R} \cos \frac{3mx}{R} - 4 \cos \frac{2mx}{R} \cos \frac{4mx}{R} \\
& + 3 \cos \frac{mx}{R} \cos \frac{4mx}{R} \cos \frac{mx}{R} - 4 \cos \frac{4mx}{R} \cos \frac{5mx}{R} - 3 \cos \frac{2mx}{R} \cos \frac{4mx}{R} \cos \frac{2mx}{R} \\
& + \frac{1}{2} \cos \frac{3mx}{R} \cos \frac{4mx}{R} \cos \frac{mx}{R} - \frac{9}{2} \cos \frac{mx}{R} \cos \frac{4mx}{R} \cos \frac{3mx}{R} - \cos \frac{4mx}{R} \cos \frac{4mx}{R} \quad \Bigg\} \\
& + 3 \cos \frac{mx}{R} \cos \frac{mx}{R} - 4 \cos \frac{2mx}{R} + 3 \cos \frac{2mx}{R} \cos \frac{2mx}{R} + \frac{9}{2} \cos \frac{3mx}{R} \cos \frac{mx}{R} \\
& + \frac{1}{2} \cos \frac{mx}{R} \cos \frac{3mx}{R} + \cos \frac{4mx}{R} \quad \Bigg]
\end{aligned}$$

[illegible]

$$\begin{aligned}
 (1) &= \frac{2}{2} \left[\left(\cos \frac{m\pi}{R} - \cos \frac{3m\pi}{R} \right) \left(\cos \frac{m\pi}{R} - \cos \frac{3m\pi}{R} \right) - \left(\cos \frac{m\pi}{R} + \cos \frac{3m\pi}{R} \right) \left(\cos \frac{m\pi}{R} + \cos \frac{3m\pi}{R} \right) \right] \\
 &= \frac{2}{2} \left[-2 \cos \frac{3m\pi}{R} \cos \frac{m\pi}{R} - 2 \cos \frac{m\pi}{R} \cos \frac{3m\pi}{R} \right] = -9 \left[\cos \frac{3m\pi}{R} \cos \frac{m\pi}{R} + \cos \frac{m\pi}{R} \cos \frac{3m\pi}{R} \right]
 \end{aligned}$$

$$(2) = \frac{3}{4} \left[3 \left(\cos \frac{2m\pi}{R} - \cos \frac{4m\pi}{R} \right) \left(1 - \cos \frac{2m\pi}{R} \right) - 5 \left(\cos \frac{2m\pi}{R} + \cos \frac{4m\pi}{R} \right) \left(1 + \cos \frac{2m\pi}{R} \right) \right]$$

$$= \frac{3}{4} \left[-2 \cos \frac{2m\pi}{R} - 8 \cos \frac{4m\pi}{R} - 8 \cos \frac{2m\pi}{R} \cos \frac{2m\pi}{R} - 2 \cos \frac{4m\pi}{R} \cos \frac{2m\pi}{R} \right]$$

$$= -\frac{3}{2} \left[\cos \frac{2m\pi}{R} + 4 \cos \frac{4m\pi}{R} + i \cos \frac{2m\pi}{R} \cos \frac{2m\pi}{R} + \cos \frac{4m\pi}{R} \cos \frac{2m\pi}{R} \right]$$

$$(3) = -\frac{3}{2} \left[\cos \frac{2m\pi}{R} + 4 \cos \frac{4m\pi}{R} + 4 \cos \frac{2m\pi}{R} \cos \frac{2m\pi}{R} + \cos \frac{2m\pi}{R} \cos \frac{4m\pi}{R} \right]$$

$$(4) = \frac{3}{4} \left[3 \left(\cos \frac{m\pi}{R} - \cos \frac{5m\pi}{R} \right) \left(\cos \frac{m\pi}{R} - \cos \frac{3m\pi}{R} \right) - 5 \left(\cos \frac{m\pi}{R} + \cos \frac{5m\pi}{R} \right) \left(\cos \frac{m\pi}{R} + \cos \frac{3m\pi}{R} \right) \right]$$

$$= \frac{3}{4} \left[-2 \cos \frac{m\pi}{R} \cos \frac{m\pi}{R} - 8 \cos \frac{5m\pi}{R} \cos \frac{m\pi}{R} - 8 \cos \frac{m\pi}{R} \cos \frac{3m\pi}{R} - 2 \cos \frac{5m\pi}{R} \cos \frac{3m\pi}{R} \right]$$

$$= -\frac{3}{2} \left[\cos \frac{m\pi}{R} \cos \frac{m\pi}{R} + 4 \cos \frac{5m\pi}{R} \cos \frac{m\pi}{R} + 4 \cos \frac{m\pi}{R} \cos \frac{3m\pi}{R} + \cos \frac{5m\pi}{R} \cos \frac{3m\pi}{R} \right]$$

$$(5) = -\frac{3}{2} \left[\cos \frac{m\pi}{R} \cos \frac{m\pi}{R} + 4 \cos \frac{m\pi}{R} \cos \frac{5m\pi}{R} + i \cos \frac{3m\pi}{R} \cos \frac{m\pi}{R} + \cos \frac{3m\pi}{R} \cos \frac{5m\pi}{R} \right]$$

$$\begin{aligned}
 (6) &= \frac{g}{4} \left[\left(\cos \frac{2m\pi}{R} - \cos \frac{4m\pi}{R} \right) \left(\cos \frac{2m\pi}{R} - \cos \frac{4m\pi}{R} \right) - \left(\cos \frac{2m\pi}{R} + \cos \frac{4m\pi}{R} \right) \left(\cos \frac{2m\pi}{R} + \cos \frac{4m\pi}{R} \right) \right] \\
 &= -\frac{g}{2} \left[\cos \frac{4m\pi}{R} \cos \frac{2m\pi}{R} + \cos \frac{2m\pi}{R} \cos \frac{4m\pi}{R} \right]
 \end{aligned}$$

$$\begin{aligned}
& -\frac{1}{4} \cos \frac{m\pi}{R} \cos \frac{m\pi}{R} - \frac{1}{4} \cos \frac{7m\pi}{R} \cos \frac{m\pi}{R} - \frac{1}{4} \cos \frac{3m\pi}{R} \cos \frac{3m\pi}{R} - \frac{1}{4} \cos \frac{5m\pi}{R} \cos \frac{3m\pi}{R} \\
& - \cos \frac{4m\pi}{R} \cos \frac{4m\pi}{R} \left\{ + 3 \cos \frac{m\pi}{R} \cos \frac{m\pi}{R} + 4 \cos \frac{2m\pi}{R} + 3 \cos \frac{2m\pi}{R} \cos \frac{2m\pi}{R} + \frac{9}{2} \cos \frac{3m\pi}{R} \cos \frac{3m\pi}{R} \right. \\
& \left. + \frac{1}{2} \cos \frac{m\pi}{R} \cos \frac{3m\pi}{R} + \cos \frac{4m\pi}{R} \right\}
\end{aligned}$$

$$\begin{aligned}
\Delta F &= \left(\frac{m}{R}\right)^2 E \frac{f_1}{f} \\
&\left[\cos \frac{2\pi x}{D} \left\{ \left(-\frac{f_1 m^2}{f}\right) \left(\frac{9}{2} + \frac{9}{8} + \frac{3}{2} + 6\right) + 4 \right\} \right. \\
&\quad + \cos \frac{2\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f}\right) \left(\frac{9}{2} + \frac{9}{8} + \frac{3}{2} + 6\right) \right\} \\
&\quad + \cos \frac{4\pi x}{R} \left\{ \left(-\frac{f_1 m^2}{f}\right) \left(\frac{9}{2} + 6\right) + 1 \right\} \\
&\quad + \cos \frac{4\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f}\right) \left(\frac{9}{2} + 6\right) \right\} \\
&\quad + \cos \frac{6\pi x}{R} \left\{ \left(-\frac{f_1 m^2}{f}\right) \left(\frac{9}{8}\right) \right\} \\
&\quad + \cos \frac{6\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f}\right) \left(\frac{9}{8}\right) \right\} \\
&\quad + \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f}\right) \left(\frac{3}{2} + \frac{3}{2} + 6 + 1 + \frac{1}{4} + 6 + 1 + \frac{1}{4}\right) + 3 \right\} \\
&\quad + \cos \frac{2\pi x}{R} \cos \frac{2\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f}\right) \left(6 + 6 + 16 + \frac{3}{2} + 16 + \frac{3}{2}\right) + 3 \right\} \\
&\quad + \cos \frac{\pi x}{R} \cos \frac{3\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f}\right) \left(7 + 6 + 6 + \frac{3}{2} + 9\right) + \frac{1}{2} \right\} \\
&\quad + \cos \frac{3\pi x}{R} \cos \frac{\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f}\right) \left(9 + 6 + 9 + 6 + \frac{3}{2}\right) + \frac{9}{2} \right\} \\
&\quad + \cos \frac{3\pi x}{R} \cos \frac{3\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f}\right) \left(9 + \frac{9}{4} + 9 + \frac{9}{4}\right) \right\} \\
&\quad + \cos \frac{4\pi x}{R} \cos \frac{2\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f}\right) \left(\frac{3}{2} + \frac{9}{2} + 4 + 6 + 4\right) \right\} \\
&\quad + \cos \frac{2\pi x}{R} \cos \frac{4\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f}\right) \left(\frac{3}{2} + \frac{9}{2} + 6 + 4 + 4\right) \right\}
\end{aligned}$$

$$\begin{aligned}
& + \cos \frac{5\pi x}{R} \cos \frac{\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f} \right) \left(6 + 1 + \frac{3}{2} \right) \right\} \\
& + \cos \frac{\pi x}{R} \cos \frac{5\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f} \right) \left(6 + 1 + \frac{3}{2} \right) \right\} \\
& + \cos \frac{3\pi x}{R} \cos \frac{5\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f} \right) \left(\frac{3}{2} + \frac{9}{4} \right) \right\} \\
& + \cos \frac{5\pi x}{R} \cos \frac{3\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f} \right) \left(\frac{3}{2} + \frac{9}{4} \right) \right\} \\
& + \cos \frac{6\pi x}{R} \cos \frac{2\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f} \right) \left(\frac{3}{2} \right) \right\} \\
& + \cos \frac{2\pi x}{R} \cos \frac{6\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f} \right) \left(\frac{3}{2} \right) \right\} \\
& + \cos \frac{\pi x}{R} \cos \frac{7\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f} \right) \left(\frac{1}{4} \right) \right\} \\
& + \cos \frac{7\pi x}{R} \cos \frac{\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f} \right) \left(\frac{1}{4} \right) \right\} \\
& + \cos \frac{4\pi x}{R} \cos \frac{4\pi y}{R} \left\{ \left(-\frac{f_1 m^2}{f} \right) (1+1) \right\}
\end{aligned}$$

$$\begin{aligned}
F = & \left(\frac{R}{m} \right)^2 \left[-\frac{1}{16} \left(\frac{165}{64} f_1 m^2 - 4 \right) \cos \frac{2m\lambda}{R} - \frac{1}{16} \frac{105}{64} f_1 m^2 \cos \frac{2m\lambda}{R} - \frac{1}{256} \left(\frac{21}{16} f_1 m^2 - 1 \right) \cos \frac{4m\lambda}{R} \right. \\
& - \frac{1}{256} \frac{21}{16} f_1 m^2 \cos \frac{4m\lambda}{R} - \frac{1}{1296} \frac{9}{64} f_1 m^2 \cos \frac{6m\lambda}{R} - \frac{1}{1296} \frac{9}{64} f_1 m^2 \cos \frac{6m\lambda}{R} - \frac{1}{4} \left(\frac{25}{16} f_1 m^2 - 3 \right) \cos \frac{2m\lambda}{R} \cos \frac{m\lambda}{R} \\
& - \frac{1}{64} \left(\frac{47}{8} f_1 m^2 - 3 \right) \cos \frac{2m\lambda}{R} \cos \frac{2m\lambda}{R} - \frac{1}{100} \left(\frac{63}{16} f_1 m^2 - \frac{1}{2} \right) \cos \frac{m\lambda}{R} \cos \frac{3m\lambda}{R} - \frac{1}{100} \left(\frac{63}{16} f_1 m^2 - \frac{1}{2} \right) \cos \frac{3m\lambda}{R} \cos \frac{m\lambda}{R} \\
& - \frac{1}{324} \frac{45}{16} f_1 m^2 \cos \frac{3m\lambda}{R} \cos \frac{3m\lambda}{R} - \frac{1}{400} \frac{5}{2} f_1 m^2 \cos \frac{4m\lambda}{R} \cos \frac{2m\lambda}{R} - \frac{1}{400} \frac{5}{2} f_1 m^2 \cos \frac{2m\lambda}{R} \cos \frac{4m\lambda}{R} \\
& - \frac{1}{676} \frac{17}{16} f_1 m^2 \cos \frac{5m\lambda}{R} \cos \frac{m\lambda}{R} - \frac{1}{676} \frac{17}{16} f_1 m^2 \cos \frac{m\lambda}{R} \cos \frac{5m\lambda}{R} - \frac{1}{1156} \frac{15}{32} f_1 m^2 \cos \frac{5m\lambda}{R} \cos \frac{3m\lambda}{R} \\
& - \frac{1}{1156} \frac{15}{32} f_1 m^2 \cos \frac{3m\lambda}{R} \cos \frac{5m\lambda}{R} - \frac{1}{1600} \frac{3}{16} f_1 m^2 \cos \frac{6m\lambda}{R} \cos \frac{2m\lambda}{R} - \frac{1}{1600} \frac{3}{16} f_1 m^2 \cos \frac{2m\lambda}{R} \cos \frac{6m\lambda}{R} \\
& - \frac{1}{2500} \frac{1}{32} f_1 m^2 \cos \frac{7m\lambda}{R} \cos \frac{m\lambda}{R} - \frac{1}{2500} \frac{1}{32} f_1 m^2 \cos \frac{m\lambda}{R} \cos \frac{7m\lambda}{R} - \frac{1}{1024} \frac{1}{4} f_1 m^2 \cos \frac{4m\lambda}{R} \cos \frac{4m\lambda}{R} \left. \right] \\
& - \frac{5}{2} \rho^2 + \frac{1}{2} \chi^2
\end{aligned}$$

$$\begin{aligned}
\sigma_z^2 = E \frac{p}{\rho} \Bigg[& + \frac{1}{4} \frac{105}{64} \rho^2 m^2 \cos \frac{2m\lambda}{R} + \frac{1}{16} \frac{21}{16} \rho^2 m^2 \cos \frac{4m\lambda}{R} + \frac{1}{36} \frac{9}{144} \rho^2 m^2 \cos \frac{6m\lambda}{R} \\
& + \frac{1}{4} \left(\frac{35}{16} \rho^2 m^2 - 3 \right) \cos \frac{m\lambda}{R} \cos \frac{m\lambda}{R} + \frac{1}{16} \left(\frac{47}{8} \rho^2 m^2 - 3 \right) \cos \frac{2m\lambda}{R} \cos \frac{2m\lambda}{R} + \frac{9}{100} \left(\frac{63}{16} \rho^2 m^2 - \frac{1}{2} \right) \cos \frac{3m\lambda}{R} \cos \frac{3m\lambda}{R} \\
& + \frac{1}{100} \left(\frac{63}{16} \rho^2 m^2 - \frac{1}{2} \right) \cos \frac{3m\lambda}{R} \cos \frac{m\lambda}{R} + \frac{9}{324} \frac{45}{16} \rho^2 m^2 \cos \frac{3m\lambda}{R} \cos \frac{3m\lambda}{R} + \frac{4}{400} \frac{5}{2} \rho^2 m^2 \cos \frac{4m\lambda}{R} \cos \frac{2m\lambda}{R} \\
& + \frac{16}{400} \frac{5}{2} \rho^2 m^2 \cos \frac{2m\lambda}{R} \cos \frac{4m\lambda}{R} + \frac{1}{676} \frac{17}{16} \rho^2 m^2 \cos \frac{5m\lambda}{R} \cos \frac{m\lambda}{R} + \frac{25}{876} \frac{17}{16} \rho^2 m^2 \cos \frac{m\lambda}{R} \cos \frac{5m\lambda}{R} \\
& + \frac{25}{1156} \frac{15}{32} \rho^2 m^2 \cos \frac{3m\lambda}{R} \cos \frac{5m\lambda}{R} + \frac{9}{1156} \frac{15}{32} \rho^2 m^2 \cos \frac{5m\lambda}{R} \cos \frac{3m\lambda}{R} + \frac{4}{1600} \frac{3}{16} \rho^2 m^2 \cos \frac{6m\lambda}{R} \cos \frac{2m\lambda}{R} \\
& + \frac{36}{1600} \frac{3}{16} \rho^2 m^2 \cos \frac{2m\lambda}{R} \cos \frac{6m\lambda}{R} + \frac{49}{2500} \frac{1}{32} \rho^2 m^2 \cos \frac{m\lambda}{R} \cos \frac{7m\lambda}{R} + \frac{1}{2500} \frac{1}{32} \rho^2 m^2 \cos \frac{7m\lambda}{R} \cos \frac{m\lambda}{R} \\
& + \frac{16}{1024} \frac{1}{4} \rho^2 m^2 \cos \frac{4m\lambda}{R} \cos \frac{4m\lambda}{R} \Bigg] - \sigma
\end{aligned}$$

$$\begin{aligned}
-\frac{1}{2} \left(\frac{\partial u}{\partial y} \right)^2 = & -\frac{1}{2} \frac{1}{\rho} m^2 \left(\frac{p}{\rho} \right)^2 \left[\frac{9}{4} \left(1 + \cos \frac{2m\lambda}{R} \right) \left(1 - \cos \frac{2m\lambda}{R} \right) + 2 - 2 \cos \frac{4m\lambda}{R} + \frac{9}{16} \left(1 + \cos \frac{2m\lambda}{R} \right) \left(1 - \cos \frac{4m\lambda}{R} \right) \right. \\
& + \frac{1}{16} \left(1 + \cos \frac{6m\lambda}{R} \right) \left(1 - \cos \frac{2m\lambda}{R} \right) + \frac{9}{16} \left(1 + \cos \frac{2m\lambda}{R} \right) \left(1 - \cos \frac{6m\lambda}{R} \right) + \frac{1}{32} - \frac{1}{32} \cos \frac{f m \lambda}{R} \\
& + 6 \cos \frac{m\lambda}{R} \left(\cos \frac{m\lambda}{R} - \cos \frac{3m\lambda}{R} \right) + \frac{9}{4} \left(\cos \frac{m\lambda}{R} + \cos \frac{3m\lambda}{R} \right) \left(\cos \frac{m\lambda}{R} - \cos \frac{3m\lambda}{R} \right) + \frac{3}{4} \left(\cos \frac{2m\lambda}{R} + \cos \frac{4m\lambda}{R} \right) \left(1 - \cos \frac{2m\lambda}{R} \right) \\
& \left. + \frac{9}{4} \left(1 + \cos \frac{2m\lambda}{R} \right) \left(\cos \frac{2m\lambda}{R} - \cos \frac{4m\lambda}{R} \right) + \frac{3}{4} \cos \frac{2m\lambda}{R} \left(\cos \frac{3m\lambda}{R} - \cos \frac{5m\lambda}{R} \right) + \dots \right]
\end{aligned}$$

$$-\frac{1}{2} \left(\frac{\partial \psi}{\partial y} \right)^2 = -\frac{1}{2} m^2 \left(\frac{f_1}{\rho} \right)^2 \left[\frac{9}{4} + 2 + \frac{9}{16} + \frac{1}{16} + \frac{9}{16} + \frac{1}{32} + \frac{1}{32} + \dots \right]$$

$$= -\frac{1}{2} m^2 \left(\frac{f_1}{\rho} \right)^2 \frac{175}{32} + \dots$$

$$\left(\frac{A}{E} + v \frac{\sigma}{E} \right) - \frac{175}{4096} \rho_1 (f_1 m^2) + (f_0 + \frac{9}{64} f_1) = 0$$

$$\frac{A}{E} = \frac{175}{4096} \rho_1 (f_1 m^2) - (f_0 + \frac{9}{64} f_1) - v \frac{\sigma}{E}$$

$$\rho_1 = -8 \frac{\sigma}{E} \left[\left(\frac{\sigma}{E} + v \frac{A}{E} \right) + \frac{175}{4096} \rho_1 (f_1 m^2) \right]$$

$$\rho_1 = -4 \frac{\sigma}{E} \left[2(1-v^2) \frac{\sigma}{E} + \frac{175}{2048} (1+v) \rho_1 (f_1 m^2) - 2v(f_0 + \frac{9}{64} f_1) \right]$$

$$4 \left[\left(\frac{\sigma}{E} \right)^2 + \left(\frac{A}{E} \right)^2 + 2v \frac{\sigma A}{E} \right]$$

$$= 4 \left[\left(\frac{\sigma}{E} \right)^2 + \frac{175^2}{4096} \rho_1^2 (f_1 m^2)^2 + (f_0 + \frac{9}{64} f_1)^2 + v^2 \left(\frac{\sigma}{E} \right)^2 - \frac{175}{2048} \rho_1 (f_1 m^2) (f_0 + \frac{9}{64} f_1) \right]$$

$$- \frac{175}{2048} v \frac{\sigma}{E} \rho_1 (f_1 m^2) + 2v \frac{\sigma}{E} (f_0 + \frac{9}{64} f_1) + v \frac{\sigma}{E} \frac{175}{2048} \rho_1 (f_1 m^2) - 2v \frac{\sigma}{E} (f_0 + \frac{9}{64} f_1) - 2v^2 \left(\frac{\sigma}{E} \right)^2$$

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$$\begin{aligned}
 f_1 + 4 \left[\left(\frac{\sigma}{E} \right)^2 + 2\nu \frac{\sigma}{E} \frac{\epsilon}{E} \right] &= K \\
 &= -4 \left[(1-\nu^2) \left(\frac{\sigma}{E} \right)^2 + \frac{175}{2048} (1+\nu) f_1' \left(\frac{\sigma}{E} \right) \frac{\sigma}{E} - 2\nu \left(\epsilon_0 + \frac{9}{64} f_1' \right) \frac{\sigma}{E} - \frac{175^2}{4096} f_1'^2 \left(\frac{\sigma}{E} \right)^2 - \left(\epsilon_0 + \frac{9}{64} f_1' \right)^2 \right. \\
 &\quad \left. + \frac{175}{2048} f_1' \left(\frac{\sigma}{E} \right)^2 \left(\epsilon_0 + \frac{9}{64} f_1' \right) \right]
 \end{aligned}$$

$$\frac{\partial K}{\partial \epsilon_0} = 0,$$

$$\boxed{-2\nu \frac{\sigma}{E} - 2 \left(\epsilon_0 + \frac{9}{64} f_1' \right) + \frac{175}{2048} f_1' \left(\frac{\sigma}{E} \right)^2 = 0}$$

$$K = -4 \left[(1-\nu^2) \left(\frac{\sigma}{E} \right)^2 + \frac{175}{2048} (1+\nu) f_1' \left(\frac{\sigma}{E} \right) \frac{\sigma}{E} - \frac{175^2}{4096} f_1'^2 \left(\frac{\sigma}{E} \right)^2 + \left(\epsilon_0 + \frac{9}{64} f_1' \right)^2 \right]$$

$$\boxed{K = -4 \left(\frac{\sigma}{E} \right)^2 - \frac{175}{512} \left(\frac{\sigma}{E} \right) f_1' \left(\frac{\sigma}{E} \right)^2}$$

$$\begin{aligned}
\rho_2 - \text{Constant Part} &= \frac{\rho_1^2}{64} \left[(f_1' m^2)^2 \left(\frac{1}{8} \frac{105^2}{64^2} + \frac{1}{128} \frac{21^2}{16^2} + \frac{1}{648} \frac{81}{84^2} + \frac{1}{8} \frac{105^2}{64^2} + \frac{1}{128} \frac{21^2}{16^2} + \frac{1}{648} \frac{81}{84^2} \right. \right. \\
&+ \frac{1}{4} \frac{35^2}{16^2} + \frac{1}{64} \frac{47^2}{64} + \frac{1}{100} \frac{63^2}{16^2} + \frac{1}{324} \frac{16^2}{16^2} + \frac{1}{400} \frac{25}{4} + \frac{1}{676} \frac{17^2}{16^2} + \frac{1}{676} \frac{12^2}{16^2} \\
&+ \frac{2}{1156} \frac{225}{32^2} + \frac{2}{1600} \frac{9}{16^2} + \frac{2}{2500} \frac{1}{32^2} + \frac{1}{1024} \frac{1}{16} \left. \right) - (f_1' m^2) \left(\frac{1}{8} \frac{105}{8} + \frac{1}{128} \frac{21}{8} + \frac{1}{648} \frac{105}{8} + \frac{1}{128} \frac{21}{8} + \frac{1}{648} \frac{105}{8} + \frac{1}{64} \frac{141}{4} \right. \\
&+ \frac{1}{100} \frac{63}{16} + \frac{1}{100} \frac{63}{16} \left. \right) + \left(2 + \frac{1}{128} + \frac{9}{4} + \frac{9}{64} + \frac{1}{200} \right) \Bigg] \\
&= \frac{\rho_1^2}{64} \left[(f_1' m^2)^2 \left(\frac{1}{4} \frac{105^2}{64^2} + \frac{1}{64} \frac{21^2}{16^2} + \frac{1}{324} \frac{81}{64^2} + \frac{1}{4} \frac{35^2}{16^2} + \frac{1}{64} \frac{47^2}{64} + \frac{1}{50} \frac{63^2}{16^2} + \frac{1}{324} \frac{45^2}{16^2} \right. \right. \\
&+ \frac{1}{32} + \frac{1}{338} \frac{17^2}{16^2} + \frac{1}{578} \frac{225}{32^2} + \frac{1}{800} \frac{9}{16^2} + \frac{1}{1250} \frac{1}{32^2} + \frac{1}{1024} \frac{1}{16} \left. \right) \\
&- \frac{1}{64} (f_1' m^2) \left(105 + \frac{21}{16} + 210 + \frac{141}{4} + \frac{126}{25} \right) + \left(2 + \frac{1}{128} + \frac{9}{4} + \frac{9}{64} + \frac{1}{200} \right) \Bigg] \\
&= \frac{\rho_1^2}{64} \left[\frac{1}{64^2} (f_1' m^2)^2 \left(2756.25 + 110.25 + 0.25 + 4900 + 2209 + 127008 + 100 + 128 + 13680 \right. \right. \\
&+ 1.557 + 0.18 + 0.003 + 0.25 \left. \right) - \frac{1}{64} (f_1' m^2) (315 + 13125 + 35.25 + 504) \\
&+ (2 + 2.3984 + 0.005) \Bigg] \\
&= \frac{\rho_1^2}{64} \left[2.80505 (f_1' m^2)^2 - 5.5719 (f_1' m^2) + 4.4034 \right]
\end{aligned}$$

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$$P_3 = \frac{1}{12(1-\nu^2)} \left(\frac{1}{R}\right)^2 m^4 \left[36 + 32 + 32 + 36 + 25 + 25 + 2 \right]$$

$$= \frac{1}{12(1-\nu^2)} \times \frac{1}{84} \times \frac{47}{16} m^4 \left(\frac{1}{R}\right)^2 r_1^2 = \frac{47}{192(1-\nu^2)} \left(\frac{1}{R}\right)^2 m^4 r_1^2$$

Total potential

$$-4 \left(\frac{\sigma}{E}\right)^2 - \frac{175}{512} \left(\frac{\sigma}{E}\right)^2 r_1^2 m^2 + \frac{r_1^2}{64} \left[2.60505 \left(\frac{r_1}{m}\right)^2 - 5.57191 \left(\frac{r_1}{m}\right)^3 + 4.4034 \right] + \frac{47}{192(1-\nu^2)} \left(\frac{1}{R}\right)^2 m^4 r_1^2$$

$$\therefore \frac{175}{256} \left(\frac{\sigma}{E}\right)^2 m^2 = \frac{1}{32} \left[5.61010 \left(\frac{r_1}{m}\right)^2 - 8.35787 \left(\frac{r_1}{m}\right)^3 + 4.4034 \right] + \frac{47}{96(1-\nu^2)} \left(\frac{1}{R}\right)^2 m^4$$

$$\frac{\sigma}{E} = \left[0.25646 r_1^2 + 0.78702 \left(\frac{1}{R}\right)^2 \right] m^2 - 0.38207 r_1^2 + 0.20130 \frac{1}{m^2}$$

$$\frac{\sigma}{E} R = \left[0.20650 \left(\frac{r}{E}\right)^2 + 0.63370 \right]^{\frac{1}{2}} - 0.38207 \left(\frac{r}{E}\right)$$

$$\left\{ (0.20650)^2 - 0.38207^2 \times 0.20650 \right\} \left(\frac{r}{E}\right)^2 = 0.38207^2 \times 0.63370$$

$$\left(\frac{r}{E}\right)^2 = \frac{0.38207^2 \times 0.63370}{0.20650 \times 0.6052} = 10.471 \times \frac{0.14598}{0.20650}$$

6.2

$$\frac{\psi}{R} = (f_0 + \frac{1}{4}f_1) + \frac{1}{2}f_1 \left[\cos \frac{mX}{R} \cos \frac{mY}{R} + \frac{1}{4} \cos \frac{2mX}{R} + \frac{1}{4} \cos \frac{2mY}{R} \right] \quad \underline{628}$$

$$+ \frac{1}{2}f_2 \left[\cos \frac{mX}{R} + \cos \frac{mY}{R} \right]$$

$$\left(\frac{\partial \psi}{\partial y} \right) = -m \left[\frac{1}{2}f_1 \left\{ \cos \frac{mX}{R} \sin \frac{mY}{R} + \frac{1}{2} \sin \frac{2mY}{R} \right\} + \frac{1}{2}f_2 \sin \frac{mY}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 \psi}{\partial x^2} = - \left(\frac{m}{R} \right)^2 \left\{ \frac{1}{2}f_1 \left[\cos \frac{mX}{R} \cos \frac{mY}{R} + \cos \frac{2mY}{R} \right] + \frac{1}{2}f_2 \cos \frac{mX}{R} \right\}$$

$$\frac{1}{R} \frac{\partial^2 \psi}{\partial y^2} = - \left(\frac{m}{R} \right)^2 \left\{ \frac{1}{2}f_1 \left[\cos \frac{mX}{R} \cos \frac{mY}{R} + \cos \frac{2mY}{R} \right] + \frac{1}{2}f_2 \cos \frac{mY}{R} \right\}$$

$$\frac{1}{R} \frac{\partial^2 \psi}{\partial x \partial y} = \left(\frac{m}{R} \right)^2 \left\{ \frac{1}{2}f_1 \sin \frac{mX}{R} \sin \frac{mY}{R} \right\}$$

$$\Delta \Delta F = E \left(\frac{m}{R} \right)^2 \left[\left(\frac{1}{2}f_1 m \right)^2 - \frac{1}{2} \cos \frac{2mX}{R} - \frac{1}{2} \cos \frac{2mY}{R} \right]$$

$$- \frac{1}{2} \left(\cos \frac{mX}{R} + \cos \frac{3mX}{R} \right) \cos \frac{mY}{R} - \frac{1}{2} \cos \frac{mX}{R} \left(\cos \frac{mY}{R} + \cos \frac{3mY}{R} \right) - \cos \frac{2mX}{R} \cos \frac{2mY}{R} \Bigg\}$$

$$+ \frac{1}{4}f_1 f_2 m^2 \left\{ - \frac{1}{2} \cos \frac{mX}{R} \left(1 + \cos \frac{2mY}{R} \right) - \frac{1}{2} \left(1 + \cos \frac{2mX}{R} \right) \cos \frac{mY}{R} \right.$$

$$\left. - \cos \frac{2mX}{R} \cos \frac{mY}{R} - \cos \frac{mX}{R} \cos \frac{2mY}{R} \right\} - \frac{1}{4}f_2^2 m^2 \cos \frac{mX}{R} \cos \frac{mY}{R}$$

$$+ \frac{1}{2}f_1 \left\{ \cos \frac{mX}{R} \cos \frac{mY}{R} + \cos \frac{2mX}{R} \right\} + \frac{1}{2}f_2 \cos \frac{mX}{R} \Bigg]$$

$$\Delta F = E\left(\frac{m^2}{R}\right) \left[\frac{1}{2} f_2 \left(1 - \frac{1}{4} f_1 m^2 \right) \cos \frac{mR}{R} - \frac{1}{8} f_1 f_2 m^2 \cos \frac{mR}{R} + \frac{1}{2} f_1 \left(1 - \frac{1}{4} f_1 m^2 \right) \cos \frac{2mR}{R} \right. \\ \left. - \frac{1}{8} f_1^2 m^2 \cos \frac{2mR}{R} + \left(\frac{1}{2} f_1 - \frac{1}{4} f_1^2 m^2 - \frac{1}{4} f_2 m^2 \right) \cos \frac{mR}{R} - \frac{3}{8} f_1 f_2 m^2 \cos \frac{2mR}{R} \right. \\ \left. - \frac{3}{8} f_1 f_2 m^2 \cos \frac{mR}{R} - \frac{1}{8} f_1^2 m^2 \cos \frac{3mR}{R} - \frac{1}{8} f_1^2 m^2 \cos \frac{mR}{R} - \frac{1}{4} f_1^2 m^2 \cos \frac{3mR}{R} - \frac{1}{4} f_1^2 m^2 \cos \frac{2mR}{R} \right]$$

$$F = E\left(\frac{R}{m}\right)^2 \frac{1}{2} \left[f_2 \left(1 - \frac{1}{4} f_1 m^2 \right) \cos \frac{mR}{R} - \frac{1}{4} f_1 f_2 m^2 \cos \frac{mR}{R} + \frac{1}{16} f_1 \left(1 - \frac{1}{4} f_1 m^2 \right) \cos \frac{2mR}{R} \right.$$

$$\left. - \frac{1}{16} f_1^2 m^2 \cos \frac{2mR}{R} + \frac{1}{4} \left(f_1 - \frac{1}{2} f_1^2 m^2 - \frac{1}{2} f_2 m^2 \right) \cos \frac{mR}{R} - \frac{1}{25} f_1 f_2 m^2 \cos \frac{2mR}{R} \right.$$

$$\left. - \frac{1}{25} f_1 f_2 m^2 \cos \frac{mR}{R} - \frac{1}{100} f_1^2 m^2 \cos \frac{3mR}{R} - \frac{1}{100} f_1^2 m^2 \cos \frac{mR}{R} - \frac{1}{100} f_1^2 m^2 \cos \frac{2mR}{R} \right]$$

$$\tilde{G}_2 + \tilde{G}_4 = (-5 + \lambda) + E \frac{1}{2} \left[f_2 \left(\frac{1}{4} f_1 m^2 - 1 \right) \cos \frac{mR}{R} + \frac{1}{4} f_1 f_2 m^2 \cos \frac{mR}{R} + \frac{1}{4} f_1 \left(\frac{1}{4} f_1 m^2 - 1 \right) \cos \frac{2mR}{R} \right.$$

$$\left. + \frac{1}{16} f_1^2 m^2 \cos \frac{2mR}{R} + \frac{1}{2} \left(\frac{1}{2} f_1^2 m^2 + \frac{1}{2} f_2 m^2 - f_1 \right) \cos \frac{mR}{R} \cos \frac{mR}{R} + \frac{3}{20} f_1 f_2 m^2 \cos \frac{2mR}{R} \right.$$

$$\left. + \frac{3}{20} f_1 f_2 m^2 \cos \frac{mR}{R} + \frac{1}{40} f_1^2 m^2 \cos \frac{3mR}{R} + \frac{1}{40} f_1^2 m^2 \cos \frac{mR}{R} + \frac{1}{16} f_1^2 m^2 \cos \frac{2mR}{R} \right]$$

$$-\frac{1}{2} \left(\frac{\partial \psi}{\partial x} \right)^2 = -\frac{1}{2} m^2 \frac{1}{4} \left[\frac{1}{4} \left\{ \frac{1}{4} (1 + c_0 \frac{2m^2}{R}) (1 - c_0 \frac{2m^2}{R}) + \frac{1}{2} c_0 \frac{2m^2}{R} / c_0 \frac{m^2}{R} - c_0 \frac{3m^2}{R} \right\} \right.$$

$$\left. + \frac{1}{2} (1 - c_0 \frac{4m^2}{R}) \left(\frac{1}{2} \rho_2^2 (1 - c_0 \frac{2m^2}{R}) + \dots \right) \right]$$

$$= -\frac{1}{8} m^2 \left[\frac{3}{8} \rho_1^2 + \frac{1}{8} \rho_2^2 \right] + \dots$$

$$\frac{1}{E} + 4 \frac{\sigma}{E} - \frac{3}{16} \rho_1^2 m^2 - \frac{1}{16} \rho_2^2 m^2 + (\rho_0 + \frac{1}{4} \rho) = 0$$

$$\boxed{\frac{1}{E} = \frac{3}{8} \rho_1^2 m^2 + \frac{1}{16} \rho_2^2 m^2 - (\rho_0 + \frac{1}{4} \rho) - 4 \frac{\sigma}{E}}$$

$$\rho_1 = -8 \frac{\sigma}{E} \left[\frac{\sigma}{E} + 4 \frac{1}{E} + \frac{3}{64} \rho_1^2 m^2 + \frac{1}{16} \rho_2^2 m^2 \right]$$

$$= -4 \left[2(1 - \nu^2) \left(\frac{\sigma}{E} \right)^2 + (1 + \nu) \frac{3}{32} \frac{\sigma}{E} \rho_1^2 m^2 + (1 + \nu) \frac{1}{8} \frac{\sigma}{E} \rho_2^2 m^2 - 2\nu (\rho_0 + \frac{1}{4} \rho) \right]$$

$$4 \left[\left(\frac{\sigma}{E} \right)^2 + \left(\frac{1}{E} \right)^2 + 2\nu \frac{\sigma}{E} \frac{1}{E} \right] = 4 \left[(1 + \nu^2) \left(\frac{\sigma}{E} \right)^2 + \frac{9}{16 \cdot 96} \rho_1^4 m^4 + \frac{1}{256} \rho_2^4 m^4 + (\rho_0 + \frac{1}{4} \rho)^2 + \frac{3}{512} \rho_2^2 \rho_1^2 m^4 \right.$$

$$\left. - \frac{3}{32} \rho_1^2 m^2 (\rho_0 + \frac{1}{4} \rho) - \frac{3}{32} \frac{4 \sigma}{E} \rho_1^2 m^2 - \frac{1}{8} \rho_2^2 m^2 (\rho_0 + \frac{1}{4} \rho) - \frac{1}{8} \frac{4 \sigma}{E} \rho_2^2 m^2 + 2\nu \frac{\sigma}{E} (\rho_0 + \frac{1}{4} \rho) \right]$$

$$+ \frac{3}{32} \frac{4 \sigma}{E} \rho_1^2 m^2 + \frac{1}{8} \frac{4 \sigma}{E} \rho_2^2 m^2 - 2\nu \frac{\sigma}{E} (\rho_0 + \frac{1}{4} \rho) - 2\nu \left(\frac{\sigma}{E} \right)^2 \left. \right]$$

$$= 4 \left[(1-v^2) \left(\frac{\sigma}{E} \right)^2 + \frac{9}{4096} \rho_1^4 m^4 + \frac{1}{256} \rho_2^4 m^4 + (\rho_0 + \frac{1}{4} \rho_1)^2 + \frac{3}{512} \rho_2^2 m^2 - \left(\frac{3}{32} \rho_1^2 m^2 + \frac{1}{8} \rho_2^2 m^2 \right) \left(\rho_0 + \frac{1}{4} \rho_1 \right) \right]$$

$$K = -4 \left[(1-v^2) \left(\frac{\sigma}{E} \right)^2 + (1+\nu) \frac{3}{32} \frac{\sigma}{E} \rho_1^2 m^2 + (1+\nu) \frac{1}{8} \frac{\sigma}{E} \rho_2^2 m^2 - 2\nu \left(\rho_0 + \frac{1}{4} \rho_1 \right) \frac{\sigma}{E} \right]$$

$$- \frac{9}{4096} \rho_1^4 m^4 - \frac{1}{256} \rho_2^4 m^4 - \left(\rho_0 + \frac{1}{4} \rho_1 \right)^2 - \frac{3}{512} \rho_1^2 m^2 + \left(\frac{3}{32} \rho_1^2 m^2 + \frac{1}{8} \rho_2^2 m^2 \right) \left(\rho_0 + \frac{1}{4} \rho_1 \right) \right]$$

$$\frac{\partial K}{\partial \rho_0} = 0$$

$$-2\nu \frac{\sigma}{E} \left(\rho_0 + \frac{1}{4} \rho_1 \right) + \left(\frac{3}{32} \rho_1^2 m^2 + \frac{1}{8} \rho_2^2 m^2 \right) \frac{\sigma}{E} = 0$$

$$K = -4 \left[(1-v^2) \left(\frac{\sigma}{E} \right)^2 + (1+\nu) \frac{3}{32} \frac{\sigma}{E} \rho_1^2 m^2 + (1+\nu) \frac{1}{8} \frac{\sigma}{E} \rho_2^2 m^2 - \frac{9}{4096} \rho_1^4 m^4 - \frac{1}{256} \rho_2^4 m^4 - \frac{3}{512} \rho_2^2 m^2 \left(\rho_0 + \frac{1}{4} \rho_1 \right)^2 \right]$$

$$= -4 \left[(1-v^2) \left(\frac{\sigma}{E} \right)^2 + \frac{3}{32} \frac{\sigma}{E} \rho_1^2 m^2 + \frac{1}{8} \frac{\sigma}{E} \rho_2^2 m^2 \right]$$

$$K = -4 \left(\frac{\sigma}{E} \right)^2 - \frac{3}{8} \frac{\sigma}{E} \rho_1^2 m^2 - \frac{1}{2} \frac{\sigma}{E} \rho_2^2 m^2$$

$$\rho_2 = \frac{1}{4} \left[2 \left(\frac{1}{4} \rho_1^2 m^2 - 1 \right) \rho_2^2 + \frac{1}{8} \rho_2^3 m^4 + \frac{1}{8} \rho_1^2 \left(\frac{1}{4} \rho_1^2 m^2 - 1 \right)^2 + \frac{1}{128} \rho_1^4 m^4 + \frac{1}{4} \left(\frac{1}{2} \rho_1^2 + \frac{1}{2} \rho_2^2 m^2 \right)^2 \right] \\ + \frac{16}{400} \rho_1^2 m^2 + \frac{1}{100} \rho_1^4 m^4 + \frac{1}{256} \rho_1^4 m^4$$

$$= \frac{1}{4} \left[\frac{1}{8} \rho_1^2 m^2 - \frac{1}{16} \rho_1^2 m^2 + \frac{1}{8} \rho_2^2 + \frac{1}{8} \rho_2^2 m^2 + \frac{1}{128} \rho_1^4 m^4 - \frac{1}{16} \rho_1^4 m^4 + \frac{1}{8} \rho_1^2 + \frac{1}{8} \rho_2^2 + \frac{1}{128} \rho_1^4 m^4 \right] \\ + \frac{1}{16} \rho_1^4 m^4 + \frac{1}{16} \rho_1^4 m^4 + \frac{1}{8} \rho_1^2 + \frac{1}{8} \rho_2^2 + \frac{1}{128} \rho_1^4 m^4 - \frac{1}{16} \rho_1^4 m^4 + \frac{1}{8} \rho_1^2 m^2 + \frac{1}{8} \rho_2^2 m^2 + \frac{1}{200} \rho_1^2 m^4 \\ + \frac{1}{800} \rho_1^4 m^4 + \frac{1}{256} \rho_1^4 m^4$$

$$= \frac{1}{4} \left[\rho_1^4 m^4 \left[\frac{1}{800} + \frac{1}{256} + \frac{1}{128} + \frac{1}{128} \cdot \frac{1}{16} \right] + \rho_1^2 m^2 \left[\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{200} \right] + \rho_2^4 m^4 \left[\frac{1}{16} \right] \right] \\ - \rho_1^3 m^2 \left(\frac{1}{16} + \frac{1}{4} \right) - \rho_1^2 m^2 (0) - \rho_1^2 m^2 \left(1 + \frac{1}{4} \right) + \frac{3}{8} \rho_1^2 + 2 \rho_2^2$$

$$\rho_2 = \frac{1}{4} \left[\frac{533}{6400} \rho_1^4 m^4 + \frac{21}{50} \rho_1^2 m^2 + \frac{1}{16} \rho_2^4 m^4 - \frac{5}{16} \rho_1^3 m^2 - \frac{5}{4} \rho_1^2 m^2 + \frac{3}{8} \rho_1^2 + 2 \rho_2^2 \right]$$

$$\rho_3 = \frac{1}{12(1-v^2)} \left(\frac{1}{R} \right) \frac{\rho_1^2 m^4}{4} \left[(4 + 2 + 2) \rho_1^2 + (2 + 2) \rho_2^2 \right] = \frac{1}{12(1-v^2)} \left(\frac{1}{R} \right) m^4 \left[2 \rho_1^2 + \rho_2^2 \right]$$

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$$\frac{3}{4} \frac{\sigma_m^2}{E} = \frac{1}{4} \left[\frac{533}{1600} f_1^2 m^4 + \frac{21}{25} f_2^2 m^4 - \frac{15}{16} f_1 m^2 - \frac{5}{4} \left(\frac{f_2}{f_1} \right) f_2 m^2 + \frac{3}{4} \right] + \frac{1}{3(1-\nu)} \left(\frac{f_2}{R} \right) m^4$$

$$\frac{\sigma_m^2}{E} = \frac{1}{4} \left[\frac{21}{25} f_1^2 m^4 + \frac{1}{4} f_2^2 m^4 - \frac{5}{2} f_1 m^2 + 4 \right] + \frac{1}{6(1-\nu)} \left(\frac{f_2}{R} \right) m^4$$

$$\frac{\sigma_m^2}{E} = \frac{533}{4800} f_1^2 m^4 + \frac{2}{25} f_1^2 m^4 \rho^2 - \frac{5}{16} f_1 m^2 - \frac{5}{12} f_1 m^2 \rho^2 + \frac{1}{4} + \frac{1}{9(1-\nu)} \left(\frac{f_2}{R} \right) m^4$$

$$\frac{\sigma_m^2}{E} = \frac{21}{100} f_1^2 m^4 + \frac{1}{16} f_1^2 m^4 \rho^2 - \frac{5}{8} f_1 m^2 + 1 + \frac{1}{6(1-\nu)} \left(\frac{f_2}{R} \right) m^4$$

$$\frac{\sigma_m^2}{E} = \left(\frac{7}{25} f_1^2 m^4 - \frac{1}{12} f_1 m^2 \right) \rho^2 + \frac{533}{4800} f_1^2 m^4 - \frac{5}{16} f_1 m^2 + \frac{1}{4} + \frac{1}{9(1-\nu)} \left(\frac{f_2}{R} \right) m^4$$

$$\frac{\sigma_m^2}{E} = \left(\frac{1}{16} f_1^2 m^4 \right) \rho^2 + \frac{21}{100} f_1^2 m^4 - \frac{5}{8} f_1 m^2 + 1 + \frac{1}{6(1-\nu)} \left(\frac{f_2}{R} \right) m^4$$

$$\begin{aligned} \left(\frac{81}{400} f_1^2 m^4 - \frac{5}{12} f_1 m^2 \right) \frac{\sigma_m^2}{E} &= \left(\frac{7}{25} f_1^2 m^4 - \frac{5}{12} f_1 m^2 \right) \left(\frac{21}{100} f_1^2 m^4 - \frac{5}{8} f_1 m^2 + 1 \right) \\ &- \frac{1}{16} f_1^2 m^4 \left(\frac{533}{4800} f_1^2 m^4 - \frac{5}{16} f_1 m^2 + \frac{1}{4} \right) + \frac{1}{3(1-\nu)} \left(\frac{f_2}{R} \right) m^4 \left(\frac{12}{300} f_1^2 m^4 - \frac{5}{24} f_1 m^2 \right) \end{aligned}$$

$$\rho = \frac{f_2}{f_1}$$

$$\left(\frac{87}{100} \rho_1 m^2 - \frac{5}{3}\right) \frac{\sigma}{E} m^2 = \left(\frac{7}{25} \rho_1 m^2 - 1\right) \left(\frac{2}{25} \rho_1 m^2 - \frac{5}{2} \rho_1 m^2 + 4\right) - \frac{1}{4} \rho_1 m^2 \left(\frac{533}{4800} \rho_1 m^4 - \frac{5}{16} \rho_1 m^2 + \frac{1}{6}\right) + \frac{1}{3(1-\nu)} \left(\frac{1}{R}\right)^2 m^4 \left(\frac{17}{75} \rho_1 m^2 - \frac{5}{6}\right)$$

$$\frac{147}{625} \rho_1^3 m^6 - \frac{35}{50} \rho_1^2 m^4 + \frac{28}{25} \rho_1 m^2 - \frac{42}{50} \rho_1^2 m^4 + \frac{5}{2} \rho_1 m^2 - 4$$

$$- \frac{533}{19200} \rho_1^3 m^6 + \frac{5}{64} \rho_1^2 m^4 - \frac{1}{16} \rho_1 m^2$$

$$\left(\frac{87}{100} \rho_1 m^2 - \frac{5}{3}\right) \frac{\sigma}{E} m^2 = \left\{ \frac{99571}{480000} \rho_1^3 m^6 - \frac{2339}{1600} \rho_1^2 m^4 + \frac{1423}{400} \rho_1 m^2 - 4 \right\} + \frac{1}{3(1-\nu)} \left(\frac{1}{R}\right)^2 m^4 \left(\frac{17}{25} \rho_1 m^2 - \frac{5}{6}\right)$$

$$\left(\frac{87}{100} \left(\frac{\rho}{E}\right) \rho - \frac{5}{3}\right) \rho = \left\{ \frac{99571}{480000} \left(\frac{\rho}{E}\right)^3 \rho^3 - \frac{2339}{1600} \left(\frac{\rho}{E}\right)^2 \rho^2 + \frac{1423}{400} \left(\frac{\rho}{E}\right) \rho - 4 \right\} + \frac{1}{3(1-\nu)} \rho^2 \left(\frac{17}{25} \left(\frac{\rho}{E}\right) \rho - \frac{5}{6}\right)$$

where $\rho = \left(m^2 \frac{\rho}{R}\right)$, Part $\left(\frac{\rho}{E}\right) = \eta$

$$m = \sqrt{\rho \frac{R}{E}}$$

634

$$\left(\frac{\sigma_R}{Et}\right) = \frac{\left\{ \frac{99571}{480000} \eta^3 \rho^3 - \frac{2339}{1600} \eta^2 \rho^2 + \frac{1423}{400} \eta \rho - 4 \right\} + \frac{1}{3(1-\nu^2)} \rho^2 \left(\frac{12}{25} \eta \rho - \frac{5}{6} \right)}{\eta \left(\frac{\rho^2}{100} \eta \rho - \frac{5}{3} \right)}$$

$$\left(\frac{\rho^2}{100} \eta \rho - \frac{5}{3} \right) \left\{ \frac{99571}{160000} \eta^3 \rho^3 - \frac{2339}{800} \eta^2 \rho^2 + \frac{1423}{400} \eta \rho + \frac{1}{3(1-\nu^2)} \rho^2 \left(\frac{12}{25} \eta \rho - \frac{5}{6} \right) \right\}$$

$$- \left(\frac{\rho^2}{50} \eta \rho - \frac{5}{3} \right) \left\{ \frac{99571}{480000} \eta^3 \rho^3 - \frac{2339}{1600} \eta^2 \rho^2 + \frac{1423}{400} \eta \rho - 4 + \frac{1}{3(1-\nu^2)} \rho^2 \left(\frac{12}{25} \eta \rho - \frac{5}{6} \right) \right\} = 0$$

$$\frac{\rho^2}{100} \eta \rho \left\{ \frac{99571}{480000} \eta^3 \rho^3 - 0 - \frac{1423}{400} \eta \rho + 8 + \frac{1}{3(1-\nu^2)} \rho^2 \left(\frac{12}{25} \eta \rho \right) \right\}$$

$$- \frac{5}{3} \left\{ \frac{99571}{240000} \eta^3 \rho^3 - \frac{2339}{1600} \eta^2 \rho^2 + 4 + \frac{1}{3(1-\nu^2)} \rho^2 \left(\frac{34}{25} \eta \rho - \frac{5}{6} \right) \right\} = 0$$

$$\frac{8662677}{48000000} \eta^4 \rho^4 - \frac{123801}{40000} \eta^2 \rho^2 + \frac{696}{100} \eta \rho + \frac{1}{3(1-\nu^2)} \rho^2 \left(\frac{1479}{2500} \eta^2 \rho^2 \right) \left\{ \right.$$

$$\left. - \left\{ \frac{99571}{144000} \eta^3 \rho^3 - \frac{2339}{960} \eta^2 \rho^2 + \frac{20}{3} + \frac{1}{3(1-\nu^2)} \rho^2 \left(\frac{34}{45} \eta \rho - \frac{25}{18} \right) \right\} = 0 \right.$$

$$0 = 0.180472 \eta^4 \rho^4 - 0.691465 \eta^3 \rho^3 - 0.658567 \eta^2 \rho^2 + 6.96 \eta \rho - 6.666666 + \frac{1}{2.73} \rho^2 (0.1972 \eta^2 \rho^2 - 0.455555 \eta \rho + 1.366667)$$

635

$$(0.180472 \eta^4 + 0.0729344 \eta^2) \eta^4 - (0.691465 \eta^3 + 0.276260 \eta) \eta^3$$

$$- (0.658567 \eta^2 - 0.508751) \eta^2 + (6.96 \eta) \eta - 6.66667 = 0$$

$$1811.94 \eta^4 - 694.233 \eta^3 - 66.3655 \eta^2 + 69.6 \eta - 6.66667 = 0$$

$$F(\eta) = \eta^4 - 0.383144 \eta^3 - 0.0366268 \eta^2 + 0.0384119 \eta - 0.00367930 = 0$$

$$F'(\eta) = 4\eta^3 - 1.149432 \eta^2 - 0.0732536 \eta + 0.0384119$$

$$F(0.124) = +0.00002651$$

1375

$$F'(0.124) = 0.01928$$

$$F(0.122625) = -0.00000016$$

$$F(0.122633) = 0$$

$$\eta = 0.122633$$

$$G(\eta) = \eta^3 - 0.260511 \eta^2 - 0.0685740 \eta + 0.0300025 = 0$$

1376

$$G'(\eta) = 3\eta^2 - 0.521022 \eta - 0.0685740$$

$$G'(\eta) = 0 = \eta^2 - 0.173674 \eta - 0.0228580$$

$$\eta = 0.086837 \pm \sqrt{(0.086837)^2 + 0.0228580}$$

$$= 0.086837 \pm \sqrt{0.0303987}$$

$$= 0.086837 \pm 0.174352 = 0.261189$$

$$- 0.087515$$

$$\left(\frac{DR}{EL}\right) = \frac{\left\{ 0.207440(\eta\gamma)^3 - 1.461875(\eta\gamma)^2 + 3.557500(\eta\gamma) - 4 \right\} + \gamma \left\{ 0.083048(\eta\gamma) - 0.305250 \right\}}{\gamma \left(0.870000(\eta\gamma) - 1.666667 \right)}$$

$$= \frac{\left\{ 0.207440 \times 1.84426 - 1.461875 \times 1.50389 + 3.557500 \times 1.22633 - 4 \right\} - 0.0150389 \times 0.20343}{-0.122633 \times 0.599760}$$

$$= \frac{1.45632}{0.073550}$$

20 !!!

Calculation is incorrect

$$\frac{1}{6} - \frac{4}{9} = \frac{1}{3} \left(\frac{1}{2} - \frac{4}{3} \right)$$

$$\left\{ \frac{87}{400}(\eta\gamma)^2 - \frac{5}{12}\eta\gamma \right\} S^2 = \frac{475}{4800}(\eta\gamma)^2 - \frac{5}{16}(\eta\gamma) + \frac{3}{4} - \frac{5}{18(1-\nu)}\gamma^2$$

3-8

$$S^2 = \frac{0.09895833(\eta\gamma)^2 - 0.3125(\eta\gamma) + 0.75 - 0.101750\gamma^2}{(\eta\gamma) \left\{ 0.87(\eta\gamma) - 0.4166667 \right\}}$$

$$= \frac{0.514064}{0.797409} = 0.644668$$

$$S = 0.802912$$

[G e n e r a l I n f o r m a t i o n]

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